

Tutorial: Ramsey theory in Forcing

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RIMS, 2016

Outline of Tutorial

- Day 1
- ① Introduction to topological Ramsey spaces
 - ② Classes of new topological Ramsey spaces which are dense in σ -closed forcings yielding ultrafilters with complete combinatorics
- Day 2
- ① Canonical Ramsey theory for equivalence relations on fronts
 - ② Applications to exact Tukey and Rudin-Keisler structures
- Day 3
- ① Topological Ramsey spaces of strong trees
 - ② Applications to finding finite Ramsey degrees for universal relational structures, including the universal triangle-free graph

National Science Foundation Grants DMS-1301665 and DMS-1600781 and Simons Foundation Grant 245286 sponsored much of Dobrinen's research in this tutorial.

Graphics by Timothy Trujillo.

Day 1 Overview

- 1 Introduction to infinite dimensional Ramsey theory
- 2 The Ellentuck space
- 3 Abstract Topological Ramsey Spaces
- 4 Connections with forcing and ultrafilters
- 5 New topological Ramsey spaces dense inside σ -closed forcings which add ultrafilters satisfying weak partition properties
- 6 A new Ramsey theorem motivated by this study

Ramsey's Theorem and Higher Dimensional Versions

Thm. (Ramsey) Given $n, l \geq 1$ and a coloring $c : [\omega]^n \rightarrow l$, there is an infinite set $M \subseteq \omega$ such that c is monochromatic on $[M]^n$.

Ramsey's Theorem can be extended to clopen sets on the Baire space.

Def. A set $\mathcal{F} \subseteq [\omega]^{<\omega}$ is **Nash-Williams** if or $a, b \in \mathcal{F}$, $a \not\sqsubset b$.

Thm. (Nash-Williams) Every Nash-Williams set $\mathcal{F} \subseteq [\omega]^{<\omega}$ is Ramsey: Given a coloring c on a front \mathcal{F} into 2 colors, there is an $M \in [\omega]^\omega$ such that c is monochromatic on $\mathcal{F}|M := \{a \in \mathcal{F} : a \subseteq M\}$.

Extensions of Ramsey's Theorem to higher dimensions

The Nash-Williams Theorem was later extended by Galvin and Prikry to all metrically Borel subsets of the Baire space.

Silver extended it to all metrically analytic subsets of the Baire space.

The optimal extension of Ramsey's Theorem to infinite dimensions is Ellentuck's Theorem.

This theorem uses a topology on the Baire space which refines the metric topology.

Ellentuck Space $([\omega]^\omega, \subseteq, r)$

Basis for topology: $[a, X] = \{Y \in [\omega]^\omega : a \sqsubset Y \subseteq X\}$.

This is a refinement of the metric topology on the Baire space.

Def. $\mathcal{X} \subseteq [\omega]^\omega$ is **Ramsey** iff for each $[a, X]$, there is $a \sqsubset Y \subseteq X$ such that either $[a, Y] \subseteq \mathcal{X}$ or $[a, Y] \cap \mathcal{X} = \emptyset$.

Thm. (Ellentuck) Every $\mathcal{X} \subseteq [\omega]^\omega$ with the property of Baire is Ramsey, and every meager set is Ramsey null.

Connection with Mathias Forcing

Mathias forcing \mathbb{M} has conditions $\langle a, X \rangle$, where $a \in [\omega]^{<\omega}$, $X \in [\omega]^\omega$, and $\max(a) < \min(X)$.

$\langle b, Y \rangle \leq \langle a, X \rangle$ iff $b \supseteq a$, $Y \subseteq X$, and $b \setminus a \subseteq X$.

Mathias forcing is equivalent to forcing using the basic open sets in the Ellentuck space, ordered by \subseteq .

Connections with Forcing and Ultrafilters

Def. An ultrafilter \mathcal{U} on ω is **Ramsey** if given any coloring $c : [\omega]^n \rightarrow I$, there is a $U \in \mathcal{U}$ which is homogenous for c .

$\mathcal{P}(\omega)/fin$, or equivalently, $([\omega]^\omega, \subseteq^*)$, forces a Ramsey ultrafilter.

Ramsey ultrafilters have **complete combinatorics**. One way to state this is that if there is a supercompact cardinal in V , then any Ramsey ultrafilter in V is generic for the forcing $([\omega]^\omega, \subseteq^*)$ over the Solovay model $L(\mathbb{R})$.

For an ultrafilter \mathcal{U} , let $\mathbb{M}_{\mathcal{U}}$ denote Mathias forcing where the tails are members of \mathcal{U} .

\mathbb{M} is forcing equivalent to $\mathcal{P}(\omega)/Fin * \mathbb{M}_{\mathcal{U}}$, where \mathcal{U} is the Ramsey ultrafilter forced by $\mathcal{P}(\omega)/Fin$.

Key properties from the Ellentuck space can be abstracted to give a general notion of a **topological Ramsey space**.

Abstract Topological Ramsey Spaces (\mathcal{R}, \leq, r)

\mathcal{R} is a set. \leq is quasi-order on \mathcal{R} .

For each n , $r_n(\cdot) := r(n, \cdot)$ is a restriction map on domain \mathcal{R} giving the n -th approximation to X .

$$\mathcal{AR}_n = \{r_n(X) : X \in \mathcal{R}\} \qquad \mathcal{AR} = \bigcup_{n < \omega} \mathcal{AR}_n.$$

For $a \in \mathcal{AR}$, $Y \in \mathcal{R}$, $a \sqsubset Y$ iff $r_n(Y) = a$ for some n .

Basic open sets: $[a, X] = \{Y \in \mathcal{R} : a \sqsubset Y \leq X\}$.

The topology on \mathcal{R} generated by the basic open sets is a refinement of the ‘metric topology’ on $\prod_{n < \omega} \mathcal{AR}_n$.

The Axioms **A.1** - **A.4**

A.1 (Sequencing)

- 1 $r_0(A) = \emptyset$ for all $A \in \mathcal{R}$.
- 2 $A \neq B$ implies $r_n(A) \neq r_n(B)$ for some n .
- 3 $r_n(A) = r_m(B)$ implies $n = m$ and $r_k(A) = r_k(B)$ for all $k < n$.

A.2 (Finitization) There is a quasi-ordering \leq_{fin} on \mathcal{AR} such that

- 1 $\{a \in \mathcal{AR} : a \leq_{\text{fin}} b\}$ is finite for all $b \in \mathcal{AR}$,
- 2 $A \leq B$ iff $(\forall n)(\exists m) r_n(A) \leq_{\text{fin}} r_m(B)$,
- 3 $\forall a, b, c \in \mathcal{AR} [a \sqsubset b \wedge b \leq_{\text{fin}} c \rightarrow \exists d \sqsubset c a \leq_{\text{fin}} d]$.

$\text{depth}_B(a)$ is the least n (if it exists) such that $a \leq_{\text{fin}} r_n(B)$.
If $\text{depth}_B(a) = n$, then $[\text{depth}_B(a), B]$ denotes $[r_n(B), B]$.

A.3 (Amalgamation)

- 1 If $\text{depth}_B(a) < \infty$ then $[a, A] \neq \emptyset$ for all $A \in [\text{depth}_B(a), B]$.
- 2 $A \leq B$ and $[a, A] \neq \emptyset$ imply that there is $A' \in [\text{depth}_B(a), B]$ such that $\emptyset \neq [a, A'] \subseteq [a, A]$.

For $a \in \mathcal{AR}$, $|a|$ denotes the k such that $a \in \mathcal{AR}_k$.

If $n > |a|$, then $r_n[a, A] = \{r_n(X) : X \in [a, A]\}$.

A.4 (Pigeonhole) Given $B \in \mathcal{R}$ and $a \in \mathcal{AR}$ with $\text{depth}_B(a) < \infty$, then for any $\mathcal{O} \subseteq r_{|a|+1}[a, B]$, there is $A \in [\text{depth}_B(a), B]$ such that $r_{|a|+1}[a, A] \subseteq \mathcal{O}$ or $r_{|a|+1}[a, A] \subseteq \mathcal{O}^c$.

Abstract Ellentuck Theorem

Def. $\mathcal{X} \subseteq \mathcal{R}$ is **Ramsey** iff for each $\emptyset \neq [a, A]$, there is a $B \in [a, A]$ such that either $[a, B] \subseteq \mathcal{X}$ or $[a, B] \cap \mathcal{X} = \emptyset$.

Def. (Todorćević) A triple (\mathcal{R}, \leq, r) is a **topological Ramsey space** if every subset of \mathcal{R} with the Baire property is Ramsey, and if every meager subset of \mathcal{R} is Ramsey null.

Building on prior work of Carlson and Simpson, Todorćević proved an abstract version of Ellentuck's Theorem.

Abstract Ellentuck Thm. (Todorćević) If (\mathcal{R}, \leq, r) satisfies Axioms **A.1 - A.4** and \mathcal{R} is closed (in $\mathcal{AR}^{\mathbb{N}}$), then (\mathcal{R}, \leq, r) is a topological Ramsey space.

Topological Ramsey Spaces are useful for

Finding exact Tukey and Rudin-Keisler structures in $\beta\omega$.

Proving complete combinatorics for ultrafilters satisfying some weak partition properties, and for some other structures.

Proving finite Ramsey degrees for universal relational structures.

Showing certain ultrafilters are preserved by side-by-side Sacks forcings.

Solving problems in Banach spaces.

Streamlining proofs and creating general frameworks for certain classes of structures and results.

Motivating new Ramsey theorems.

Standard Examples of Topological Ramsey Spaces

- 1 Ellentuck space
- 2 Carlson-Simpson space of equivalence relations on ω with infinitely many equivalence classes (dual Ramsey)
- 3 Pröml-Voigt spaces of parameter words and ascending parameter words
- 4 Milliken space of block sequences $\text{FIN}_k^{[\infty]}$
- 5 Carlson's space of infinite dimensional vector spaces $F^{\mathbb{N}}$ where F is a finite field.

D. and Mijares have an example schema which encompasses (2) - (5) as special cases.

All of these spaces (except the Ellentuck space) have \leq essentially given by a composition operator and are *surjective* spaces.

Inner Topological Ramsey Spaces

Several classes of σ -closed forcings adding new ultrafilters have been shown to contain dense subsets which form topological Ramsey spaces.

Like the Ellentuck space, the partial orderings \leq are given by structure-preserving injections.

The motivation for this was to find exact Tukey and Rudin-Keisler structures of ultrafilters. We will cover that tomorrow.

We will survey several of these topological Ramsey spaces and how they were constructed from known partial orderings. First, a word about ultrafilters forced by topological Ramsey spaces.

Ultrafilters forced by topological Ramsey spaces

Mijares showed that every topological Ramsey space has an induced σ -closed partial ordering of **almost reduction** \leq^* , similarly to \subseteq^* on the Ellentuck space.

(\mathcal{R}, \leq^*) forces a maximal generic filter on \mathcal{R} .

The generic ultrafilter usually induces an ultrafilter on the base set \mathcal{AR}_1 .

Selective Coideals and Complete Combinatorics

Given a topological Ramsey space (\mathcal{R}, \leq, r) , a coideal $\mathcal{U} \subseteq \mathcal{R}$ is **selective** if for each $A \in \mathcal{U}$ and any collection $(A_a)_{a \in \mathcal{A}\mathcal{R}|A}$ of members of $\mathcal{U} \upharpoonright A$, there is a $U \in \mathcal{U}$ which diagonalizes $(A_a)_{a \in \mathcal{A}\mathcal{R}|A}$.

Forcing with (\mathcal{R}, \leq^*) adds a selective ultrafilter or coideal \mathcal{U} on \mathcal{R} .

Thm. (Mijares) each (\mathcal{R}, \leq) is forcing equivalent to forcing first with (\mathcal{R}, \leq^*) to obtain a generic ultrafilter \mathcal{U} , and then forcing with the σ -closed localized version $(\mathcal{R}_{\mathcal{U}}, \leq)$ where the tails are in \mathcal{U} .

Thm. (Di Prisco/Mijares/Nieto) In the presence of a supercompact cardinal, every selective coideal $\mathcal{U} \subseteq \mathcal{R}$ is generic for (\mathcal{R}, \leq^*) over $L(\mathbb{R})$.

Laflamme's forcing to add a weakly Ramsey ultrafilter

$\mathbb{P}_1 = ([\omega]^\omega, \leq_1)$. For $X, Y \in [\omega]^\omega$, enumerating them in increasing order and in blocks of increasing size as

$X = \langle x_1^1, x_1^2, x_2^2, x_1^3, x_2^3, x_3^3, \dots \rangle$ and $Y = \langle y_1^1, y_1^2, y_2^2, y_1^3, y_2^3, y_3^3, \dots \rangle$, then $Y \leq_1 X$ iff $\forall m \exists n$ such that $\{y_1^m, \dots, y_m^m\} \subseteq \{x_1^n, \dots, x_n^n\}$.

Note: $Y \leq_1 X \implies Y \subseteq X$.

$Y \leq_1^* X$ iff $\forall^\infty n$, the n -th block of Y is contained in some block of X .

So (\mathbb{P}_1, \leq_1^*) is like $([\omega]^\omega, \subseteq^*)$ except the partial ordering is more restrictive.

Thm. (Laflamme) (\mathbb{P}_1, \leq_1^*) forces a weakly Ramsey ultrafilter.

\mathcal{U} is **weakly Ramsey** if for each finitary coloring c of $[\omega]^2$, there is a $U \in \mathcal{U}$ for which c takes on at most two colors on $[U]^2$.

$$\mathcal{U} \rightarrow (\mathcal{U})_{k,2}^2.$$

If κ is a Mahlo cardinal and G is Levy(κ)-generic over V , then any ultrafilter \mathcal{U} on ω in $V[G]$ which is not Ramsey but is rapid and satisfies $\text{RP}(k)$ for all k is generic over $\text{HOD}(\mathbb{R})^{V[G]}$ (the original form of ‘complete combinatorics’).

Laflamme's forcing (\mathbb{P}_1, \leq_1) . Example: $Y \leq_1 X$

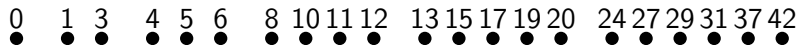


Figure: $X \in [\omega]^\omega$



Figure: $Y \in [\omega]^\omega$ & $Y \leq_1 X$

The topological Ramsey space dense in (\mathbb{P}_1, \leq_1)

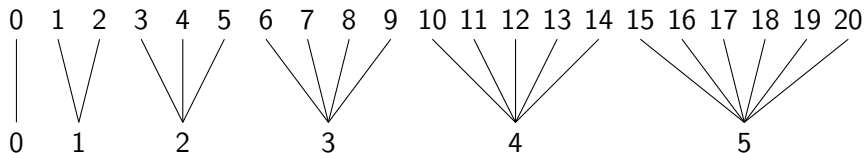


Figure: The maximum member of \mathcal{R}_1 .

The topological Ramsey space dense in (\mathbb{P}_1, \leq_1)

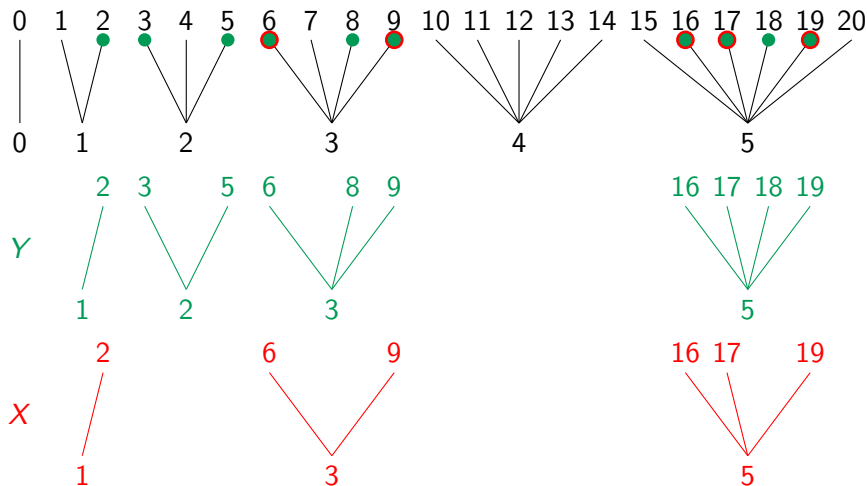


Figure: Two members X and Y of \mathcal{R}_1 with $X \leq Y$.

A subtree not in \mathcal{R}_1

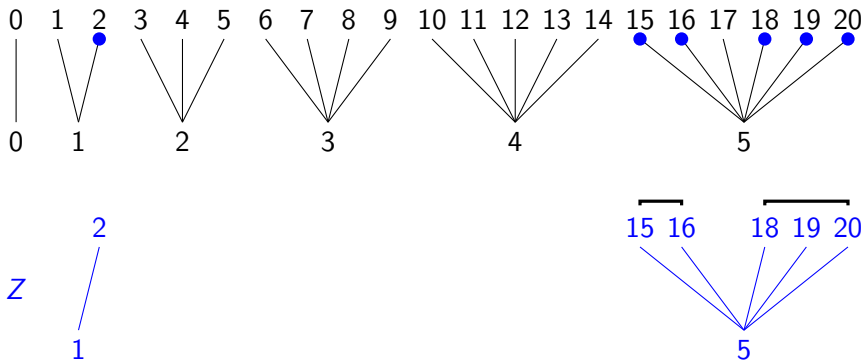


Figure: $Z \notin \mathcal{R}_1$

Thm. (D./Todorcevic) (\mathcal{R}_1, \leq) is a topological Ramsey space and is dense below any member of \mathbb{P}_1 .

The generic filter forced by (\mathcal{R}_1, \leq^*) induces an ultrafilter on \mathcal{AR}_1 . Call it \mathcal{U}_1 . \mathcal{U}_1 is **weakly Ramsey**: $\mathcal{U}_1 \rightarrow (\mathcal{U}_1)_{k,2}^2$.

Prop. For each $n \geq 2$, $\mathcal{U}_1 \rightarrow (\mathcal{U}_1)_{k,2^{n-1}}^n$.

This is stated in [Laflamme 89]. An elegant proof using topological Ramsey space methods is given in Navarro Flores' Masters Thesis.

Exercise. Prove **A.4** for the space \mathcal{R}_1 .

Ramsey spaces \mathcal{R}_α dense inside Laflamme's forcings \mathbb{P}_α , $\alpha < \omega_1$, were also constructed in [D./Todorcevic 15].

The n -square forcing of Blass

A subset of $\omega \times \omega$ of the form $s \times t$ is an n -square if $|s| = |t| = n$.

$X \subseteq \omega \times \omega$ is in $\mathbb{P}_{n\text{-square}}$ iff for each $n < \omega$, X contains an $n \times n$ -square.
For $X, Y \in \mathbb{P}_{n\text{-square}}$, $Y \leq X$ iff $Y \subseteq X$.

Thm. (Blass) $\mathbb{P}_{n\text{-square}}$ forces a \mathfrak{p} -point with two Rudin-Keisler incomparable predecessors.

Def. \mathcal{U} is a **\mathfrak{p} -point** if each sequence $X_0 \supseteq^* X_1 \supseteq^* \dots$ of members of \mathcal{U} has a pseudointersection $U \in \mathcal{U}$; i.e. $U \subseteq^* X_i$ for all i .

The Ramsey space \mathcal{H}_2 dense in the n -square forcing

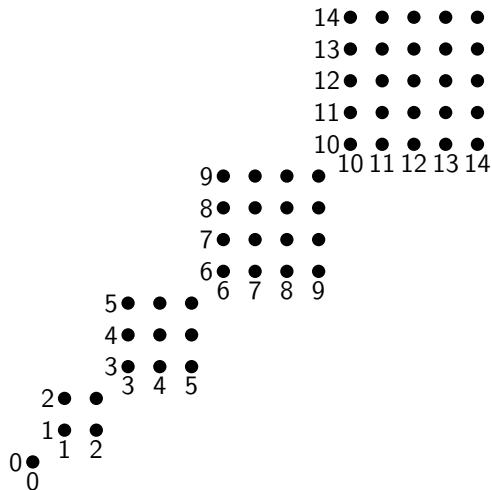


Figure: The maximum member of \mathcal{H}^2 .

The Ramsey space \mathcal{H}_2 dense in the n -square forcing

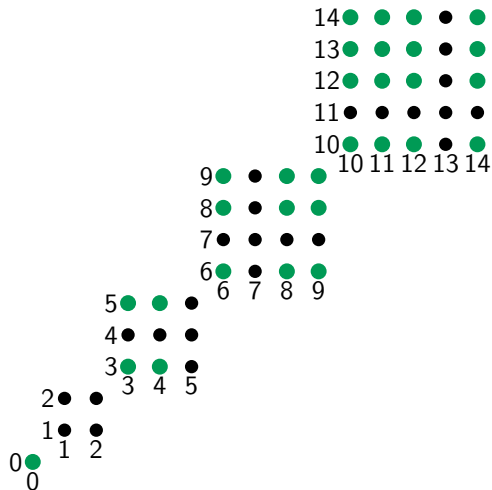


Figure: A member of X in \mathcal{H}^2 .

The Ramsey space \mathcal{H}_2 dense in the n -square forcing

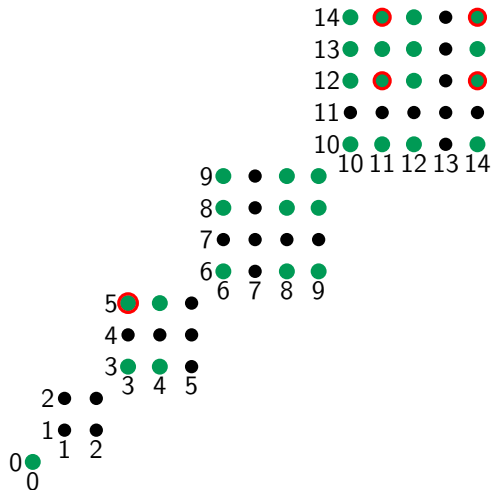


Figure: Two members Y and X of \mathcal{H}^2 with $Y \leq X$

The Hypercube topological Ramsey spaces

Thm. (D./Trujillo) (\mathcal{H}^2, \leq, r) is a topological Ramsey space.

Let \mathcal{V}_2 denote the ultrafilter forced by (\mathcal{H}^2, \leq) . Then $\mathcal{V}_2 \rightarrow (\mathcal{V}_2)_{k,6}^2$.

Rem 1. \mathcal{H}^2 behaves like a product of two copies of \mathcal{R}_1 ; each copy of \mathcal{R}_1 is recovered by projection maps.

Rem 2. Higher dimensional hypercube spaces were constructed in [D./Mijares/Trujillo] including a space where the dimension of the n -th block is $n + 1$.

Arrow Ultrafilters of Baumgartner and Taylor

Def. An ultrafilter \mathcal{U} is k -**arrow** if $\mathcal{U} \rightarrow (\mathcal{U}, k)^2$.

Thm. (Baumgartner/Taylor) There are forcings which construct ultrafilters \mathcal{W}_{k+1} which are k -arrow but not $(k+1)$ -arrow ultrafilters, for all $k \geq 2$.

Thm. (D./Mijares/Trujillo) For each $k \geq 2$, there is a topological Ramsey space \mathcal{A}_{k+1} which is dense in the Baumgartner-Taylor partial order forcing a k -arrow, not $(k+1)$ -arrow ultrafilter.

Exercise. Prove **A.4** for \mathcal{A}_3 .

The topological Ramsey space \mathcal{A}_3

Let \mathcal{K}_3 be the Fraïssé class of finite ordered triangle-free graphs.

Block structure for \mathcal{A}_3



Figure: A member X of \mathcal{A}_3

$$\text{block } n + 2 \rightarrow (\text{block } n + 1)^{\text{block } n}$$

The topological Ramsey space \mathcal{A}_3

Let \mathcal{K}_3 be the Fraïssé class of finite ordered triangle-free graphs.

Block structure for \mathcal{A}_3



Figure: Example: Two members Y and X of \mathcal{A}_3 with $Y \leq X$

$$\text{block } n + 2 \rightarrow (\text{block } n + 1)^{\text{block } n}$$

Ramsey degrees for these forced ultrafilters (in [DMT])

The space (\mathcal{A}_3, \leq^*) forces a 2-arrow but not 3-arrow ultrafilter, \mathcal{W}_3 , the same one as Baumgartner and Taylor. The Ramsey degrees are

$$\begin{aligned}\mathcal{W}_3 &\rightarrow (\mathcal{W}_3)_{k,3}^2 \\ \mathcal{W}_3 &\rightarrow (\mathcal{W}_3)_{k,12}^3 \\ \mathcal{W}_3 &\rightarrow (\mathcal{W}_3)_{k,35}^4\end{aligned}\tag{1}$$

More generally, any product \mathcal{K} of finitely many Fraïssé classes of finite ordered relational structures with the Ramsey property can be used to compose the block structures of members of a topological Ramsey space.

The topological Ramsey space structure allows one to find formulas for the numbers d such that $\mathcal{U}_{\mathcal{K}} \rightarrow (\mathcal{U}_{\mathcal{K}})_{k,d}^m$, for such forced ultrafilters.

Topological Ramsey spaces dense in creature forcings

Observation (Todorćević). There are strong connections between creature forcings and topological Ramsey spaces deserving of a systematic study.

Question. Which creature forcings are essentially topological Ramsey spaces?

In [D. TopApp 16], we proved that three of the examples of pure candidates for creature forcings given in [Roslanowski/Shelah 13] contain dense subsets which are topological Ramsey spaces.

Let \mathbf{H} be any function with $\text{dom}(\mathbf{H}) = \omega$ such that $\mathbf{H}(n)$ is a finite non-empty set for each $n < \omega$.

$$\mathcal{F}_{\mathbf{H}} = \bigcup_{u \in \text{FIN}} \prod_{n \in u} \mathbf{H}(n).$$

pure candidates are certain infinite sequences \bar{t} of creatures (finite structures). $\text{pos}(\bar{t})$ is an infinite subset of $\mathcal{F}_{\mathbf{H}}$ induced by \bar{t} .

Thm. (Rosłanowski/Shelah) (CH) There is an ultrafilter \mathcal{U} on base set $\mathcal{F}_{\mathbf{H}}$ generated by $\{\text{pos}(\bar{t}_{\alpha}) : \alpha < \omega_1\}$ for a decreasing sequence of pure candidates $\langle \bar{t}_{\alpha} : \alpha < \omega_1 \rangle$ satisfying the partition theorem:

For any \bar{t} such that $\text{pos}(\bar{t}) \in \mathcal{U}$ and any partition of $\text{pos}(\bar{t})$ into finitely many pieces, there is a pure candidate $\bar{s} \leq \bar{t}$ such that $\text{pos}(\bar{s})$ is contained in one piece of the partition and $\text{pos}(\bar{s}) \in \mathcal{U}$.

Remark 1. This generalizes the construction of a stable-ordered union ultrafilter on FIN using Hindman's Theorem.

Remark 2. The proofs in [RS] use the Galvin-Glazer method extended to certain classes of creature forcings.

Thm. (D.) Three examples of sets of pure candidates in $[RS]$ contain dense subsets forming topological Ramsey spaces.

For two of these spaces, the pigeonhole principles rely on the following product tree Ramsey theorem.

New Product Tree Ramsey Theorem

Let $\prod_{j \in n+1, p} K_j = K_0 \times \cdots \times K_{p-1} \times [K_p]^k \times K_{p+1} \times \cdots \times K_n$.

Thm. (D.) Given $k \geq 1$, a sequence of positive integers (m_0, m_1, \dots) , sets K_j , $j < \omega$ such that $|K_j| \geq j + 1$, and a coloring

$$c : \bigcup_{n < \omega} \bigcup_{p \leq n} \prod_{j \in n+1, p} K_j \rightarrow 2,$$

there are infinite sets $L, N \subseteq \omega$ such that $l_0 \leq n_0 < l_1 \leq n_1 < \dots$, and there are $H_j \subseteq K_j$, $j < \omega$, such that $|H_{l_i}| = m_i$ for each $i < \omega$, $|H_j| = 1$ for each $j \in \omega \setminus L$, and c is constant on

$$\bigcup_{n \in N} \bigcup_{l \in L \cap (n+1)} \prod_{j \in n+1, l} H_j.$$

The proof built on the following Product Tree Theorem.

Thm. (Di Prisco/Llopis/Todorcevic) There is an $R : (\mathbb{N}^+)^{<\omega} \rightarrow \mathbb{N}^+$ such that for every infinite sequence $(m_j)_{j<\omega}$ of positive integers and for every coloring

$$c : \bigcup_{n<\omega} \prod_{j \leq n} R(m_0, \dots, m_j) \rightarrow 2,$$

there exist $H_j \subseteq R(m_0, \dots, m_j)$, $|H_j| = m_j$, for $j < \omega$, such that c is constant on the product $\prod_{j \leq n} H_j$ for infinitely many $n < \omega$.

Remark. The difference is that we need sets of size k to be able to move up and down indices of the product.

Conclusion for Day 1

Topological Ramsey spaces provide a unifying framework for many ultrafilters satisfying partition properties and, moreover, yield ultrafilters with complete combinatorics. They also motivate new Ramsey theorems.

With further work (tomorrow), we obtain:

- 1 New canonical equivalence relations on fronts and barriers.
- 2 Exact Rudin-Keisler and Tukey structures as well as the structure of the Rudin-Keisler classes inside the Tukey types.

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