MATH 1953 Midterm 1

Name: ________________________________

**Instructions:** Please answer each question as completely as possible, and show all work unless otherwise indicated. You may use an approved calculator for this quiz. (Approved: non-graphing, non-programmable, doesn’t take derivatives)

1. (16 pts.) Find the equations of all tangent lines to the point (4,0) for the parametric curve given by \( x = t^2 \) and \( y = 2t^5 - 8t^3 \).
2. (25 pts.) Find the area of the shaded region trapped between the two polar equations $r = 1$ and $r = 2 \cos(\theta)$ (see picture below).
3. (16 pts.) Find the length of the polar equation \( r = \sin(\theta) + \cos(\theta) \) for \( 0 \leq \theta \leq \frac{\pi}{2} \).
4. (11 pts.) Find \( \lim_{x\to\infty} x^\frac{2}{3} \) or show that the limit does not exist.
5. (16 pts.) Does the improper integral \( \int_0^1 x \ln(x) \, dx \) converge or diverge? If it converges, find its value.
6. (8 pts. each) Tell whether the following improper integrals converge or diverge. DO NOT attempt to find the value of the integral!
(a) \( \int_{4}^{\infty} \frac{x}{x^2 + x^3 + 1} \, dx \)

(b) \( \int_{1}^{\infty} \frac{2 + \sin x}{x} \, dx \)