The February Problem of 2014:

A Gothic Arch is usually constructed by drawing two arcs around the two springing points at the two ends of a horizontal base-line usually forming the ceiling of a rectangular window. The question is that of the Radius of the two arcs if three equal sized touching circles fit into the arch above the base-line. (This pattern appears in the portal of Evans Memorial Church on DU-s Campus.) (For a more detailed Problem description see: [link](http://web.cs.du.edu/~sobieczk/mathclub/gothic/index.html))

Solution: Call the mid-points of the three equal (small) circles $A$, $B$, and $C$, where $A$ and $B$ correspond to the two circles resting on the upper window sill. The radius of the small circles is $r$. Therefore, the distance to the window sill from $A$ and $B$ is $r$.

Furthermore, the circular arc which is supposed to touch the circles around $A$ and $C$ has a point $D$, around which it is curved, just as point $E$ which we let be the midpoint of the circle (whose arc) is touching circles around $B$ and $C$.

We observe that the line perpendicular to the tangent of the arc around $D$ at the point where it is touching the circle around $C$ runs through $C$. Similarly, we see that the line perpendicular to the tangent of the arc around $D$ at the point where it is touching the circle around $A$ runs through $A$. The interception of these two perpendicular lines (through $C$ and $A$) is $D$.

Note that $D$ has to lie on the line through the upper window sill, to meet the condition that the circular arcs join the vertical sides of the window with vertical slope. Therefore, $D$ is also the interception of the horizontal line through the window sill and the bisecting line of the angle of the triangle $ABC$ at $B$. (The angle is $\pi/3$, since the three circles have the same radius.)

Note, that since a single side of $ABC$ is $2r$, and therefore, the height of the triangle $ABC$ is $\sqrt{3}r$. Denote by $F$ the point exactly between $A$ and $C$. Note, that the line between $F$ and $B$ has length $\sqrt{3}$.

The length of the line between $B$ and $D$ is $2r$. This follows from the fact that the angle between the horizontal line and the line between $B$ and $D$ is $\pi/6$. Therefore, since the distance between $A$ and the horizontal window sill line is $r$, we have $r/sin \pi/6 = 2r$ as the length between $B$ and $D$. Therefore the line between $F$ and $D$ has length $(2+\sqrt{3})r$.

Note, now, that the triangle $ADF$ has a right angle at $F$. The distance between $A$ and $F$ is $r$. Therefore the distance between $D$ and $A$ is $\sqrt{r^2 + (2+\sqrt{3})^2} = r\sqrt{8 + 4\sqrt{3}}$. Finally, the distance between $A$ and the arc around $D$ is $r$, which has to be added to length of the line between $A$ and $D$, giving $R/r = 1 + \sqrt{8 + 4\sqrt{3}}$.

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