Rigidity results for L^p-operator algebras

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Joint work with Hannes Thiel from the University of Münster

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This talk focuses mostly on group algebras (to be defined).

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Let $\lambda\colon L^1(G) \to \mathcal{B}(L^p(G))$ be the integrated form of the left regular representation: $\lambda(f)\xi = f * \xi$ for $f \in L^1(G), \xi \in L^p(G)$.

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Proof: $L^1(G)$ has a contractive approximate identity.

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When is the canonical map $F^p_{QS}(G) \to F^p(G)$ an isomorphism? Does it depend on p or on G?

We also studied analogs of $F^p_{QS}(G)$ using SL^p-spaces and QL^p-spaces.

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(I think this should be true for arbitrary G when $p \neq 2$.)

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We recover Wendel's result when $p = 1$, and with a different proof: he used extreme points of the unit ball and we used invertible isometries. Our techniques yield a stronger result, with algebras of convolvers or pseudomeasures in place of F_λ^p.

Crossed products by minimal homeomorphisms

Let $h\colon X \to X$ and $k\colon Y \to Y$ be free and minimal homeomorphisms of compact metric spaces. Their L^p-crossed products are denoted by $F^p(X, h)$ and $F^p(Y, k)$, respectively.

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Thank you.