Rethinking Connes' approach to the standard model of particle physics via non-commutative geometry.

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Overview

1. Introduction: - The NCG Standard particle model

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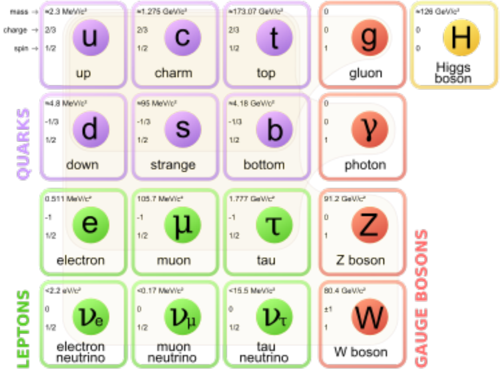
2. Reformulation: - $\{m, g\} \longleftrightarrow \{A, H, D\} \longleftrightarrow B$
3. Applications: - Constraining D_F
   - Fixing the Higgs Mass
   - Non-associative geometry
4. Open problems

Introduction: The standard model



$$S_{SM} = \int d^4x \, \overline{\psi} \not{D} \psi + \overline{\psi} \phi \psi + F^2 + (D\phi)^2 - m_\phi^2 \phi^2 + \lambda_\phi \phi^4$$

Introduction: The Standard model.

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$$S = Tr[f(D/\Lambda)] + < \psi | D\psi >$$

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   - $[a, Jb^* J^*] = 0$ for all $a, b \in A$
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3. Incorrect Higgs mass predicted

Reformulation: $\{A, H, J\} \rightarrow B_0$

$A \quad H$

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$$b^* = (a+h)^* = a^* + Jh$$

Reformulation: $\{A, H, J\} \to B_0$

Associativity:

$$
\begin{aligned}
[b_1, b_0, b_2] &= (b_1 b_0) b_2 - b_1 (b_0 b_2) = 0 \\
\to (a_1 h) a_2 - a_1 (h a_2) &= [R_{a_2}, L_{a_1}] h = 0 \\
\to [R_{a_2}, L_{a_1}] &= 0 \qquad \forall a_i \in A, h \in H, b_i \in B_0
\end{aligned}
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Involution:

$$(b_1 b_0)^* = b_0^* b_1^*$$
$$J L_{a_1} h = R_{a_1}^* J h$$
$$\rightarrow R_{a_1} = J L_{a_1}^* J^* \qquad \forall a_1 \in A, h \in H, b_i \in B_0$$

Reformulation: $\{A, H, D, J\} \to B$

$$A \to \Omega A = A \oplus \Omega^1 A \oplus \Omega^2 A \ldots$$

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$$A \to \Omega A = A \oplus \Omega^1 A \oplus \Omega^2 A ...$$

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Associativity:

$$[\omega_1, h, \omega_2] = [R_{\omega_2}, L_{\omega_1}] = 0$$
$$\to [[D, a], Jb^* J^*] = 0$$
$$\to [[D, a], J[D, b]^* J^*] = 0 ...$$

Applications:Too many Higgs Fields

$$D_F = \left(\begin{array}{cc|cc|cc|cc} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} & D_{17} & D_{18} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} & D_{27} & D_{28} \\ \hline D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} & D_{37} & D_{38} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} & D_{47} & D_{48} \\ \hline D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} & D_{57} & D_{58} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} & D_{67} & D_{68} \\ \hline D_{71} & D_{72} & D_{73} & D_{74} & D_{75} & D_{76} & D_{77} & D_{78} \\ D_{81} & D_{82} & D_{83} & D_{84} & D_{85} & D_{86} & D_{87} & D_{88} \end{array}\right)$$

Basis: $\{l_R, q_R, l_L, q_L, \bar{l}_R, \bar{q}_R, \bar{l}_L, \bar{q}_L\}$

Applications:Too many Higgs Fields

Constraints on D: $D = D^*, \{D, \gamma\} = 0, DJ = \epsilon JD, [[D, a], Jb^* J^*] = 0$

$$D_F = \left(\begin{array}{cc|cc|cc|cc}
0 & 0 & y_l^\dagger & 0 & m^\dagger & n^\dagger & 0 & 0 \\
0 & 0 & 0 & y_q^\dagger & \overline{n} & 0 & 0 & 0 \\
\hline
y_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & y_q & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
m & n^T & 0 & 0 & 0 & 0 & y_l^T & 0 \\
n & 0 & 0 & 0 & 0 & 0 & 0 & y_q^T \\
\hline
0 & 0 & 0 & 0 & \overline{y}_l & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \overline{y}_q & 0 & 0
\end{array}\right)$$

Basis:$\{l_R, q_R, l_L, q_L, \overline{l}_R, \overline{q}_R, \overline{l}_L, \overline{q}_L\}$

Too Many Higgs Fields

$$[\omega_m, h, \omega_n] = 0 \to [[D, a], J[D, b]^* J^*] = 0$$

$$m = \begin{pmatrix} a & b \\ b & 0 \end{pmatrix}, \qquad n = \begin{pmatrix} \vec{c} & \vec{d} \\ 0 & 0 \end{pmatrix}$$

Applications: Incorrect Higgs mass

$$D_F = \left(\begin{array}{cc|cc|cc|cc} 0 & 0 & y_l^\dagger & 0 & m^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & y_q^\dagger & 0 & 0 & 0 & 0 \\ \hline y_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_q & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline m & 0 & 0 & 0 & 0 & 0 & y_l^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_q^T \\ \hline 0 & 0 & 0 & 0 & \overline{y}_l & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{y}_q & 0 & 0 \end{array}\right)$$

$$m = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}, \quad y_l = \begin{pmatrix} Y_\nu \alpha & -Y_e \overline{\beta} \\ Y_\nu \beta & Y_e \overline{\alpha} \end{pmatrix}, \quad y_q = \begin{pmatrix} Y_u \alpha & -Y_d \overline{\beta} \\ Y_u \beta & Y_d \overline{\alpha} \end{pmatrix}$$

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Traditional approach: symmetries $= Aut(A)$
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$$D \to D_A = D + A + JAJ^*$$

$$y_l = \begin{pmatrix} Y_\nu \alpha & -Y_e \overline{\beta} \\ Y_\nu \beta & Y_e \overline{\alpha} \end{pmatrix} \to \begin{pmatrix} Y_\nu \phi_1 & -Y_e \overline{\phi}_2 \\ Y_\nu \phi_2 & Y_e \overline{\phi}_1 \end{pmatrix}$$

$$y_q = \begin{pmatrix} Y_u \alpha & -Y_d \overline{\beta} \\ Y_u \beta & Y_d \overline{\alpha} \end{pmatrix} \to \begin{pmatrix} Y_\nu \phi_1 & -Y_e \overline{\phi}_2 \\ Y_\nu \phi_2 & Y_e \overline{\phi}_1 \end{pmatrix}$$

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Applications: Incorrect Higgs mass

$$
\begin{aligned}
S_B = \int_M \Bigg(& \frac{48 f_4 \Lambda^4}{\pi^2} - \frac{c f_2 \Lambda^2}{\pi^2} + \frac{d f(0)}{4\pi^2} - \frac{b\pi^2}{2a^2 f(0)} {v_0}^4 \\
& + \frac{c f(0) s}{24\pi^2} - \frac{4 f_2 \Lambda^2 s}{\pi^2} - \frac{3 f(0)}{10\pi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \\
& + \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a} + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu,i} + \frac{4 f_2 \Lambda^2}{\pi^2} (\partial^\beta \eta)(\partial_\beta \eta) \\
& + \frac{1}{2} (\partial^\mu h)(\partial_\mu h) + \frac{b\pi^2}{2a^2 f(0)} \left(h^4 + 4 v_0 h^3 + 4 {v_0}^2 h^2 \right) \\
& + \frac{1}{4} {g_2}^2 (v_0 + h)^2 W^{\mu *} W_\mu + \frac{1}{8} \frac{{g_2}^2}{{c_w}^2} (v_0 + h)^2 Z^\mu Z_\mu \Bigg) \sqrt{|g|}\, d^4 x.
\end{aligned}
$$

$$\frac{1}{2} {m_h}^2 h^2 = \frac{b\pi^2}{2a^2 f(0)} 4 {v_0}^2 h^2$$

$$\frac{1}{2} {M_W}^2 W^{\mu *} W_\mu = \frac{1}{4} {g_2}^2 v_0^2 W^{\mu *} W_\mu$$

$$\frac{b\pi^2}{2a^2 f(0)} h^4 =: \frac{1}{24} \lambda h^4.$$

$${m_h}^2 = \frac{4\lambda {M_W}^2}{3 {g_2}^2}.$$

$$\frac{f(0)}{2\pi^2} {g_3}^2 = \frac{f(0)}{2\pi^2} {g_2}^2 = \frac{5 f(0)}{6\pi^2} {g_1}^2 = \frac{1}{4} \qquad \lambda = 24 \frac{b}{a^2} {g_2}^2.$$

Applications: Incorrect Higgs mass

Fused algebra approach: symmetries $= Aut(B)$
Symmetry Generators: $\delta = L_a - R_a + t$, where $a = -a^* \in A$,
$[t, L_x] = [t, R_x] = [t, J] = [t, \gamma] = 0$, and $t = -t^* \in End(H)$.

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$$D \to D_B = D + A + JAJ^* + T$$

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$$m = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} Y_M \sigma(x) & 0 \\ 0 & 0 \end{pmatrix}$$

Applications: Non-associative geometry

$$[b_1, b_0, b_2] \neq 0$$

Summary:

$$\{A, H, D\} \rightarrow B$$