

THE ELEMENTARY PARTICLE CUBE

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Abstract

Postulating that spacetime is discrete, we assume that physical space is described by a 3-dimensional cubic lattice. The corresponding symmetry group of rotations has order 24 and motivates the introduction of a cubic shaped graph with 27 vertices and 351 edges. We call this graph the elementary particle cube (EPC) and consider the vertices as tiny cells that pre-elementary particles called preons can occupy and the edges as interactions between preons. The 23 nontrivial members of the symmetry group naturally associate with the 23 basic elementary particles. We assume that each elementary particle is described by a unique subgraph of the EPC. The particular subgraph is determined by symmetry and the particle's mass. We postulate that the particle mass is a certain function of the lengths of the edges in the graph representing the particle. This correspondence between particle graphs and mass appears to be quite accurate and gives a reason why leptons and quarks come in three generations. In this way, the basic elementary particles emerge in a natural way from a few simple principles. The paper ends with a discussion of hadrons, which as in the standard model, are composite systems of quarks.

1 Introduction

This article presents an approach for studying elementary particles. Theoretical and experimental investigations in physics have identified and measured

properties of the basic elementary particles. These particles have been organized into certain categories; namely, the force mediating bosons consisting of the photons, gravitons, gluons and weak bosons W^- , Z^0 , the leptons consisting of the electron e , muon μ , tau τ and their corresponding neutrinos ν_e, ν_μ, ν_τ , the quarks, down d , up u , strange s , charm c , bottom b , top t and finally, the Higgs boson H . In this article we shall mainly consider the 23 particles consisting of the gluons, weak bosons, leptons, quarks and Higgs. Their corresponding anti-particles need not be considered because they have identical properties except for charge when electric charge is present.

Our basic assumption is that spacetime is discrete [1, 2, 3, 8, 9]. We shall not be concerned with time here, so we assume that space has the structure of a 3-dimensional cubic lattice $\mathcal{S}_3 = \mathbb{Z}^3$ where \mathbb{Z} is the set of integers. Other discrete configurations are possible [4, 5], but we choose \mathcal{S}_3 because of its simplicity. The distance between adjacent lattice points of \mathcal{S}_3 are presumably on a Planck scale of about 10^{-33}cm [8, 9]. We endow \mathcal{S}_3 with the usual Euclidean norm given by $\|v\| = \sqrt{a^2 + b^2 + c^2}$ for $v = (a, b, c) \in \mathcal{S}_3$. This provides the metric $d(u, v) = \|u - v\|$ on \mathcal{S}_3 . In this way $(\mathcal{S}_3, \|\cdot\|)$ becomes a normed module over \mathbb{Z} . The *symmetry group* \mathcal{G}'_3 on \mathcal{S}_3 is the group of linear operators (3×3 matrices) $T: \mathcal{S}_3 \rightarrow \mathcal{S}_3$ that satisfy $\|Tv\| = \|v\|$ for all $v \in \mathcal{S}_3$. The group of *physical rotations* \mathcal{G}_3 is the subgroup of \mathcal{G}'_3 consisting of operators with unit determinant. The group \mathcal{G}'_3 has order 48 and \mathcal{G}_3 has order 24 [6, 7]. We shall show in Section 2 that the 23 nontrivial elements of \mathcal{G}_3 congregate into four natural subsets that we call the lepton, quark, gluon and boson types. As they should, these types contain 6,6,8,3 elements, respectively. The rotation axes of the various types suggest the construction of a cubic shaped graph with 27 vertices and 351 edges. We call this graph the elementary particle cube (EPC) and consider the vertices as tiny cells that pre-elementary particles called preons can occupy and the occupied edges as interactions between preons.

In Sections 3 and 4 we assume that each elementary particle is described by a unique subgraph of the EPC. The particular subgraph is determined by symmetry and the particle's mass. We propose a mass formula that gives the mass of a particle as a function of the lengths of the edges in the graph representing the particle. This correspondence between particle graphs and mass appears to be quite accurate. In particular, predicted particle masses agree with experiment to within about 1% accuracy. The paper ends with a discussion of hadrons, which as in the standard model, are composite systems of quarks.

2 Symmetry

As discussed in Section 1, we have the group of symmetries \mathcal{G}'_3 and the subgroup of physical rotations \mathcal{G}_3 on \mathcal{S}_3 . It is easy to check that $\mathcal{G}'_3 = \mathcal{G}_3 \cup (-\mathcal{G}_3)$. That is, every element $Z \in \mathcal{G}'_3$ satisfies $Z \in \mathcal{G}_3$ or $-Z \in \mathcal{G}_3$. We denote the elements of \mathcal{G}_3 by A, B, \dots, X . These are orthogonal 3×3 matrices with entries 0, 1, -1 and are given by [7]

$$\begin{aligned}
 A &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\
 E &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 I &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \\
 M &= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}, \quad O = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \\
 Q &= \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad T = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \\
 U &= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad V = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad W = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
 \end{aligned}$$

We now show that various elements of \mathcal{G}_3 band together to form subsets with common properties. These common properties are order and invariant

vectors. We call the set

$$\mathcal{L} = \{A, B, C, E, G, J\}$$

the *lepton type*, the set

$$\mathcal{Q} = \{O, P, R, S, T, U\}$$

the *quark type*, the set

$$\mathcal{G} = \{K, L, M, N, Q, V, W, X\}$$

the *gluon type* and the set

$$\mathcal{B} = \{D, F, H\}$$

the *boson type*. It is easy to check that the elements of \mathcal{L} have order 4, the elements of \mathcal{Q} and \mathcal{B} have order 2 and the elements of \mathcal{G} have order 3.

Invariant vectors are eigenvectors with corresponding eigenvalue 1. To study such vectors, it is convenient to introduce the following elements of \mathcal{S}_3 .

$$\begin{aligned} \text{Level 1: } u_1 &= (0, 0, 1), u_2 = (0, 0, -1), u_3 = (0, 1, 0), u_4 = (0, -1, 0), \\ u_5 &= (1, 0, 0), u_6 = (-1, 0, 0) \end{aligned}$$

$$\begin{aligned} \text{Level 2: } v_1 &= (0, 1, 1), v_2 = (0, 1, -1), v_3 = (0, -1, 1), v_4 = (0, -1, -1), \\ v_5 &= (1, 0, 1), v_6 = (1, 0, -1), v_7 = (-1, 0, 1), v_8 = (-1, 0, -1), \\ v_9 &= (1, 1, 0), v_{10} = (1, -1, 0), v_{11} = (-1, 1, 0), v_{12} = (-1, -1, 0) \end{aligned}$$

$$\begin{aligned} \text{Level 3: } w_1 &= (1, 1, 1), w_2 = (1, -1, 1), w_3 = (1, 1, -1), w_4 = (-1, 1, 1), \\ w_5 &= (-1, -1, 1), w_6 = (-1, 1, -1), w_7 = (1, -1, -1), \\ w_8 &= (-1, -1, -1) \end{aligned}$$

The reason for the level terminology is because $\|u_j\|^2 = 1$, $\|v_j\|^2 = 2$, $\|w_j\|^2 = 3$ for all applicable j . We do not normalize these vectors because we want them to be in \mathcal{S}_3 .

Let us find the invariant vectors for some elements of \mathcal{G}_3 . For A we have

$$Au_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = u_5$$

Also, $Au_6 = u_6$ so A has two level 1 invariant vectors. It is easy to check that u_5 and u_6 are the only invariant vectors of A among the three levels. Also, D has precisely u_5 and u_6 as invariant vectors among the three levels. However, A and D are different in the sense that A has order 4 and $D = A^2$ has order 2. For P we have $Pv_2 = v_2$, $Pv_3 = v_3$ and v_2, v_3 are the only invariant vectors of P among the three levels. Finally, w_3, w_5 are the only invariant vectors of L among the three levels. In summery, we have the following.

Lemma 2.1. *Among the vectors in levels 1, 2 and 3, the elements of \mathcal{L} have order 4 and precisely two invariant vectors in level 1, the elements of \mathcal{Q} have order 2 and precisely two invariant vectors in level 2, the elements of \mathcal{G} have order 3 and precisely two invariant vectors in level 3 and the elements of \mathcal{B} have order 2 and precisely two invariant vectors in level 1.*

It may be a coincidence that not counting the trivial identity element I , there are 23 elements in \mathcal{G}_3 which happens to be the number of basic elementary particles. However, it seems unlikely that such a coincidence would carry over to the fact that the elements of \mathcal{G}_3 band together into sets with common properties

$$\mathcal{G}_3 = \mathcal{L} \cup \mathcal{Q} \cup \mathcal{G} \cup \mathcal{B}$$

and cardinalities 6, 6, 8, 3 the same size as the sets of leptons, quarks, gluons and bosons, respectively. In Section 3 we shall give other connections between these subsets of \mathcal{G}_3 and the elementary particles. The corresponding anti-particles can be treated by enlarging \mathcal{G}_3 to \mathcal{G}'_3 . If $Z \in \mathcal{G}_3$ corresponds to a certain particle, then $-Z \in \mathcal{G}'_3$ corresponds to its anti-particle [7].

The invariant vectors provide a method for visualizing the rotations in \mathcal{G}_3 . We have seen that u_5 and u_6 are invariant vectors of $A \in \mathcal{G}_3$. We call the edge u_5u_6 an *axis* and can visualize A as a rotation of $\pi/2$ radians about the axis u_5u_6 . Similarly, $D = A^2$ gives the rotation of π radians about the axis u_5u_6 . In this way the elements of \mathcal{L} are rotations of $\pi/2$ about level 1 axes and the elements of \mathcal{B} are rotations of π about level 1 axes. In a similar way, the elements of \mathcal{Q} are rotations of π about level 2 axes and elements of \mathcal{G} are rotations $2\pi/3$ about level 3 axes [7].

We call the set of invariant vectors together with the origin (or center) $c = (0,0,0)$ the *elementary particle cube* \mathcal{C}_3 . (Of course, $Zc = c$ for all $Z \in \mathcal{G}_3$.) Thus,

$$\mathcal{C}_3 = \{c, u_1, \dots, u_6, v_1, \dots, v_{12}, w_1, \dots, w_8\} \quad (2.1)$$

In this way, \mathcal{G}_3 is the rotation group for the EPC \mathcal{C}_3 . We think of \mathcal{C}_3 as the complete graph with vertices given in (2.1) and pairs of vertices such as u_2v_7 its edges. As with any graph, an edge uv is considered to be the same as an edge vu so edges are actually doubleton sets $\{u, v\}$. We see that \mathcal{C}_3 has 27 vertices and $351 = 27 \cdot 26/2$ edges.

3 Mass

We postulate that corresponding to each of the 23 elementary particles there is a unique complete subgraph of \mathcal{C}_3 . That is, if p is an elementary particle, then there is a corresponding set of vertices $\{x_1, x_2, \dots, x_n\} \subseteq \mathcal{C}_3$ and their edges $x_1x_2, x_1x_3, \dots, x_{n-1}x_n$ that form a complete subgraph of the graph \mathcal{C}_3 . We assume that the vertices represent tiny cells that are occupied by pre-elementary particles called preons and that the edges represent interactions between preons. The particular subgraph corresponding to an elementary particle will be determined by symmetry and the mass of the particle. In this approach, we view an elementary particle as having the shape of part of a cube which when its center of mass is translated to the origin c it becomes a subgraph of \mathcal{C}_3 .

We propose that the mass of a particle depends on the interactions between pairs of vertices $\{x, y\}$ in its graph; that is, edges xy in its graph. If xy is an edge of \mathcal{C}_3 , how do we find the interaction strength $s(x, y)$ of this edge? We have not found a method for deriving an equation for $s(x, y)$, but we do have some guidelines. First, $s(x, y)$ should be rotationally invariant; that is, $s(Zx, Zy) = s(x, y)$ for all $Z \in \mathcal{G}_3$. Second, it appears that the interactions could be similar to a spring force that increases with distance. Now the main quantity that is rotation invariant is the norm $\|x\|$. Denoting the level by $\ell(x) = \|x\|^2$, we postulate that $s(x, y)$ obeys the following “fourth power law”

$$\begin{aligned} s(x, y) &= [2^{n(x,y)} (\ell(x) + \ell(y))]^2 d(x, y)^4 \\ &= [2^{n(x,y)} (\|x\|^2 + \|y\|^2)]^2 \|x - y\|^4 \end{aligned} \quad (3.1)$$

where

$$n(x, y) = \begin{cases} 1 & \text{if } \ell(x) = \ell(y) = 3 \\ 0 & \text{otherwise} \end{cases}$$

Equation (3.1) states that $s(x, y)$ increases with the fourth power of the distance between x and y , increases with the levels of x and y and kicks in a factor of 4 at the highest interaction levels $\ell(x) = \ell(y) = 3$.

If xy is an edge, we say that the *interaction type* for xy is $\ell(x) - \ell(y)$ where $\ell(x) \leq \ell(y)$. For example, cu_1 has interaction type $0 - 1$, v_1v_2 has interaction type $2 - 2$ and u_3w_5 has interaction type $1 - 3$. We now compute some sample interaction strengths. Suppose we want $s(c, w_3)$. Since $n(c, w_3) = 0$, $\ell(c) = 0$, $\ell(w_3) = 3$, $d(c, w_3) = \sqrt{3}$ we have

$$s(c, w_3) = 3^2(\sqrt{3})^4 = 81$$

To compute $s(v_1, w_2)$ we have $n(v_1, w_2) = 0$, $\ell(v_1) = 2$, $\ell(w_2) = 3$ and $d(v_1, w_2) = \sqrt{5}$, Hence,

$$s(v_1, w_2) = 5^2(\sqrt{5})^4 = 625$$

Finally, to compute $s(w_1, w_2)$ we have $n(w_1, w_2) = 1$, $\ell(w_1) = \ell(w_2) = 3$ and $d(w_1, w_2) = 2$ which gives

$$s(w_1, w_2) = 2^2 \cdot 6^2 \cdot 2^4 = 2304$$

Table 1 presents the interaction strengths for all the 351 edges of \mathcal{C}_3 . The notation $c(u_1, u_2, \dots, u_6)$ means cu_1, cu_2, \dots, cu_6 .

We postulate that the mass of an elementary particle is proportional to the sum of the interaction strengths for the edges of its corresponding graph plus a self-energy term. This self-energy term has the form $3n_p$ where $n_p = 0$ if the graph for particle p has fewer than 4 vertices and n_p is the number of vertices, otherwise. To set the mass units in MeV we take the proportionality constant to be the mass of the electron 0.511 MeV except for neutrinos where we use $(0.511)/3^4$ MeV. Thus, if p is a neutrino, then its mass is given by

$$m(p) = \frac{(0.511)}{81} \left[\sum_{xy \in p} s(x, y) + 3n_p \right] \quad (3.2)$$

and otherwise,

$$m(p) = (0.511) \left[\sum_{xy \in p} s(x, y) + 3n_p \right] \quad (3.3)$$

where the notation specifies that xy is an edge in the graph of p .

Type	Strength	Edges
0 – 1	1	$c(u_1, u_2, \dots, u_6)$
0 – 2	16	$c(v_1, v_2, \dots, v_{12})$
0 – 3	81	$c(w_1, w_2, \dots, w_8)$
1 – 1	16	$u_1(u_3, u_4, u_5, u_6), u_2(u_3, u_4, u_5, u_6)$ $u_3(u_5, u_6), u_4(u_5, u_6)$
	64	u_1u_2, u_3u_4, u_5u_6
1 – 2	9	$u_1(v_1, v_3, v_5, v_7), u_2(v_2, v_4, v_6, v_8), u_3(v_1, v_2, v_9, v_{11})$ $u_4(v_3, v_4, v_{10}, v_{12}), u_5(v_5, v_6, v_9, v_{10}), u_6(v_1, v_8, v_{11}, v_{12})$
	81	$u_1(v_9, v_{10}, v_{11}, v_{12}), u_2(v_9, v_{10}, v_{11}, v_{12}), u_3(v_5, v_6, v_7, v_8)$ $u_4(v_5, v_6, v_7, v_8), u_5(v_1, v_2, v_3, v_4), u_6(v_1, v_2, v_3, v_4)$
	225	$u_1(v_2, v_4, v_6, v_8), u_2(v_1, v_3, v_5, v_7), u_3(v_3, v_4, v_{10}, v_{12})$ $u_4(v_1, v_2, v_9, v_{11}), u_5(v_7, v_8, v_{11}, v_{12}), u_6(v_5, v_6, v_9, v_{10})$
1 – 3	64	$u_1(w_1, w_2, w_4, w_5), u_2(w_3, w_6, w_7, w_8), u_3(w_1, w_3, w_4, w_6)$ $u_4(w_2, w_5, w_6, w_7), u_5(w_1, w_2, w_3, w_7), u_6(w_4, w_5, w_6, w_8)$
	576	$u_1(w_3, w_6, w_7, w_8), u_2(w_1, w_2, w_4, w_5), u_3(w_2, w_5, w_7, w_8)$ $u_4(w_1, w_3, w_4, w_8), u_5(w_4, w_5, w_6, w_8), u_6(w_1, w_2, w_3, w_7)$
2 – 2	64	$v_1(v_5, v_7, v_9, v_{11}), v_2(v_8, v_9, v_{11}), v_3(v_4, v_5, v_7, v_{10}, v_{12})$ $v_4(v_6, v_8, v_{10}, v_{12}), v_5(v_9, v_{10}), v_6(v_9, v_{10}), v_7(v_{11}, v_{12}), v_8(v_{11}, v_{12})$
	256	$v_1(v_2, v_3), v_2(v_4, v_6), v_5(v_6, v_7), v_8(v_6, v_7), v_9(v_{10}, v_{11}), v_{12}(v_{10}, v_{11})$
	576	$v_1(v_6, v_8, v_{10}, v_{12}), v_2(v_5, v_7, v_{10}, v_{12}), v_3(v_6, v_8, v_9, v_{11})$ $v_4(v_5, v_7, v_9, v_{11}), v_5(v_{11}, v_{12}), v_6(v_{11}, v_{12}), v_7(v_9, v_{10}), v_8(v_9, v_{10})$
	1024	$v_1v_4, v_2v_3, v_5v_8, v_6v_7, v_9v_{12}, v_{10}v_{11}$
2 – 3	25	$v_1(w_1, w_4), v_2(w_3, w_6), v_3(w_2, v_5), v_4(w_7, w_8), v_5(w_1, w_2), v_6(w_3, w_7)$ $v_7(w_4, w_5), v_8(w_6, w_8), v_9(w_1, w_3), v_{10}(w_2, w_7), v_{11}(w_4, w_6), v_{12}(w_5, w_8)$
	625	$v_1(w_2, w_3, w_5, w_6), v_2(w_1, w_4, w_7, w_8), v_3(w_1, w_4, w_7, w_8)$ $v_4(w_2, w_3, w_5, w_6), v_5(w_3, w_4, w_5, w_7), v_6(w_1, w_2, w_6, w_8)$ $v_7(w_1, w_2, w_6, w_8), v_8(w_3, w_4, w_5, w_7), v_9(w_2, w_4, w_6, w_7)$ $v_{10}(w_1, w_3, w_5, w_8), v_{11}(w_1, w_3, w_5, w_8), v_{12}(w_2, w_4, w_6, w_7)$
	2025	$v_1(w_7, w_8), v_2(w_2, w_5), v_3(w_3, v_6), v_4(w_1, w_4), v_5(w_6, w_8), v_6(w_4, w_5)$ $v_7(w_3, w_7), v_8(w_1, w_2), v_9(w_5, v_8), v_{10}(w_4, w_6), v_{11}(w_2, w_7), v_{12}(w_1, w_3)$
3 – 3	2304	$w_1(w_2, w_3, w_4), w_2(w_5, w_7), w_3(w_6, w_7), w_4(w_5, w_6), w_5w_8, w_6w_8, w_7w_8$
	9216	$w_1(w_5, w_6, w_7), w_2(w_3, w_4, w_8), w_3(w_4, w_8), w_4w_8, w_5(w_6, w_7), w_6w_7$
	20736	$w_1w_8, w_2w_6, w_3w_5, w_4w_7$

Table 1 (Interaction Strengths)

Equations (3.2), (3.3) and a symmetry criterion essentially determine the graph of an elementary particle. Conversely, we shall show that the graph of an elementary particle determines its mass to within about 1%. The symmetry criterion, which is motivated by Lemma 2.1, proceeds as follows. Lepton and boson graphs contain at least one level 1 vertex, quark graphs contain at least one level 2 vertex and gluon graphs contain at least one level 3 vertex.

4 Graphs

To roughly describe the graph of an elementary particle, we define the *configuration* of a complete subgraph of \mathcal{C}_3 to be a 4-tuple (a_0, a_1, a_2, a_3) where a_j is the number of vertices at level j . By convention, $c = (0, 0, 0, 0)$ is the only vertex at level 0. The configuration is invariant under the group \mathcal{G}_3 and indicates the geometry of the graph. However, it does not determine the graph uniquely. To accurately describe the graph, we must specify its vertices. Moreover, an isomorphic image under \mathcal{G}_3 will give the same mass. There are occasions when the mass formula does not uniquely determine the particle configuration. For example, the configurations $(1, 0, 1, 0)$ and $(0, 2, 0, 0)$ can both result in the mass 8.17 MeV. However, if this is to describe the down quark, then the second alternative is eliminated because a quark must have a level 2 vertex. Also, we shall see that ν_τ and μ have the same configuration but different vertices and masses.

We now list our proposed configurations for the elementary particles. A gluon is massless so there are no interactions and it must therefore have just one vertex. Since this vertex must be at level 3, the gluon configuration is $g = (0, 0, 0, 1)$. This is consistent with the fact that there are eight gluons and eight level 3 vertices. We propose that the electron neutrino has configuration $(0, 1, 0, 0)$. This again gives zero mass which is consistent with its mass being less than 2.2×10^{-6} MeV, an amount that is orders of magnitude less than the other nonzero particle masses. Our list of configurations is the following:

Neutrinos: $\nu_e = (0, 1, 0, 0)$, $\nu_\mu = (1, 1, 1, 0)$, $\nu_\tau = (1, 1, 1, 1)$

Leptons: $e = (1, 1, 0, 0)$, $\mu = (1, 1, 1, 1)$, $\tau = (1, 1, 2, 1)$

Quarks: $d = (1, 0, 1, 0)$, $u = (0, 1, 1, 0)$, $s = (1, 1, 2, 0)$, $c = (0, 1, 2, 1)$

$b = (1, 2, 2, 2)$, $t = (1, 4, 12, 8)$

Gluons: $g = (0, 0, 0, 1)$

Bosons: $X^- = (1, 4, 12, 5)$, $Z^0 = (1, 5, 12, 5)$, $H = (1, 1, 10, 7)$

Assuming these configurations, we now apply (3.2) and (3.3) for specific vertices to predict particle masses and compare them to experimental values which are given in parentheses. When considering experimental values, one must remember that neutrino masses have not been determined very precisely. This also applies to quark masses because quarks have not (cannot) be isolated so that their masses are essentially theoretical. One might argue that because of the flexibility of configurations and vertex choice, one can obtain practically any desired mass. However, with a closer examination, one can see that the mass values are fairly restricted. When applying (3.2), (3.3), we write the interaction strengths in the order of interaction type given in Table 1 followed by the $3n_p$ term.

Muon neutrino: vertices c, u_1, v_1

$$m(\nu_\mu) = \frac{(0.511)}{81} (1 + 16 + 9) = 0.164 (0.165)$$

Tau neutrino: vertices c, u_1, v_2, w_2

$$m(\nu_\mu) = \frac{(0.511)}{81} (1 + 16 + 81 + 225 + 64 + 2025 + 12) = 15.29 (15.5)$$

Electron: vertices c, u_1

$$m(e) = 0.511 (0.511)$$

Muon: vertices c, u_1, v_1, w_1

$$m(\mu) = (0.511) (1 + 16 + 81 + 9 + 64 + 25 + 12) = 106.29 (105.6)$$

Tau: vertices c, u_2, v_1, v_4, w_7

$$\begin{aligned} m(\tau) &= (0.511) (1 + 32 + 81 + 234 + 64 + 1024 + 2050 + 15) \\ &= 1789.01 (1776.8) \end{aligned}$$

Down Quark: vertices c, v_1

$$m(d) = (0.511) (16) = 8.17 (4-8)$$

Up Quark: vertices u_1, v_1

$$m(u) = (0.511) (9) = 4.599 (4-8)$$

Strange Quark: vertices c, u_1, v_1, v_9

$$m(s) = (0.511)(1 + 32 + 90 + 64 + 12) = 101.69 (101)$$

Charm Quark: vertices u_1, v_{10}, v_{11}, w_5

$$m(c) = (0.511)(162 + 64 + 1024 + 1250 + 12) = 1283.63 (1270)$$

Bottom Quark: vertices $c, u_1, u_3, v_1, v_4, w_1, w_4$

$$\begin{aligned} m(b) &= (0.511)(2 + 32 + 162 + 16 + 468 + 256 + 1024 + 4100 + 2304 + 21) \\ &= 4284.74 (4200) \end{aligned}$$

Top Quark: vertices $c, u_3, u_4, u_5, u_6, v_1, v_2, \dots, v_{12}, w_1, w_2, \dots, w_8$

$$\begin{aligned} m(t) &= (0.511)(4 + 192 + 648 + 320 + 5058 + 10240 + 24576 + 79200 + 221184 + 75) \\ &= 174505 (173800) \end{aligned}$$

X^- Boson: vertices $c, u_3, u_4, u_5, u_6, v_1, v_2, \dots, v_{12}, w_4, w_5, w_6, w_7, w_8$

$$\begin{aligned} m(X^-) &= (0.511)(4 + 192 + 405 + 192 + 5040 + 6400 + 24576 + 49500 + 69120 + 66) \\ &= 79457.9 (80,000) \end{aligned}$$

Z^0 Boson: vertices $c, u_2, u_3, \dots, u_6, v_1, v_2, \dots, v_{12}, w_3, w_4, w_5, w_7, w_8$

$$\begin{aligned} m(Z^0) &= (0.511)(5 + 192 + 405 + 256 + 6300 + 8256 + 24576 + 49500 + 87552 + 69) \\ &= 90503.7 (91,000) \end{aligned}$$

H Boson: vertices $c, u_1, v_2, v_3, v_5, v_6, \dots, v_{12}, w_2, w_3, \dots, w_8$

$$\begin{aligned} m(H) &= (0.511)(1 + 16 + 567 + 1026 + 2496 + 17600 + 58150 + 165888 + 57) \\ &= 125609 (125,000) \end{aligned}$$

We point out that because of the three levels of vertices, this explains why leptons and quarks come in three generations.

5 Composite Systems

We can apply our previous methods to determine masses of composite quark systems called hadrons, which consist of two types, the mesons and the

baryons. As in the standard model, a meson is formed from a quark-antiquark pair $q_1\bar{q}_2$ and a baryon is formed from a quark triple $q_1q_2q_3$. For the interactions between q_1 and \bar{q}_2 , we do not consider the individual preon vertices of q_1 and \bar{q}_2 but view q_1 and \bar{q}_2 as single entities described by their configurations. Similarly, for a baryon $q_1q_2q_3$ we have interactions for the pairs (q_1, q_2) , (q_1, q_3) and (q_2, q_3) .

We propose that the mass formula for mesons is:

$$m(q_1\bar{q}_2) = \frac{1}{2} [m(q_1) + m(q_2)] + (0.511) \sum s(x, y) \quad (5.1)$$

where x and y are the noncentral (non-level 0) vertices of q_1 and q_2 and $s(x, y)$ are interaction strengths between x and y given in Table 1. We also propose that the mass formula for baryons is:

$$m(q_1, q_2, q_3) = \frac{1}{3} [m(q_1) + m(q_2) + m(q_3)] + (0.511) \sum s(x, y) \quad (5.2)$$

where x, y are the noncentral vertices from pairs (q_1, q_2) , (q_1, q_3) , (q_2, q_3) and $s(x, y)$ are again certain interaction strengths between x and y given in Table 1. The number of interactions of each type is determined by the configurations and the main problem is to decide which interaction strengths to choose from the various possibilities.

We postulate that in composite systems, the u and d quarks can interchange configurations. This is analogous to the standard model where, for example, a meson can be in a superposition of $u\bar{u}$ and $d\bar{d}$ mesons. We then define

$$u' = (0, 1, 1, 0)' = (1, 0, 1, 0)$$

where u' has a d configuration, but retains mass $m(u)$ and the u electric charge ($2/3$). Similarly,

$$d' = (1, 0, 1, 0)' = (0, 1, 1, 0)$$

where d' has a u configuration, but retains a mass $m(d)$ and the d electric charge ($-1/3$).

We now consider interaction strengths. In order to limit the number of choices of interaction strengths, we first reduce the possibilities in Table 1 to two or fewer representations. These are given in Table 2 in an order that we shall find convenient.

Type	1 – 1	1 – 2	2 – 2	1 – 3	2 – 3	3 – 3
Strengths	16 (mesons) 64 (baryons)	81, 225	64, 256	64, 576	625, 2025	2304, 9216

Table 2 (Representative Strengths)

A hadron can be described by the configurations of its quark constituents. In turn, these configurations determine interactions of various types that we call the *type sequences*

$$[n_{11}(1-1), n_{12}(1-2), n_{22}(2-2), n_{13}(1-3), n_{23}(2-3), n_{33}(3-3)] \quad (5.3)$$

where n_{ij} is the number of type $(i-j)$ interactions for the quark constituents of the hadron. For example, the K^+ meson has the quark composition $K^+ = u\bar{s}$. Since u and s have configurations $(0, 1, 1, 0)$ and $(1, 1, 2, 0)$ respectively, we write

$$K^+ = u\bar{s} = (0, 1, 1, 0)(1, 1, 2, 0)$$

Considering pairs of vertices in these two configurations, we see that there are one type 1 – 1, three type 1 – 2 and two type 2 – 2 interactions. We then say that the type sequence for K^+ is

$$[1(1-1), 3(1-2), 2(2-2)] \quad (5.4)$$

For simplicity, we have omitted the zero terms from (5.3) in the expression (5.4). For a more complicated example, the Λ_c^+ baryon has the composition

$$\Lambda_c^+ = udc = (0, 1, 1, 0)(1, 0, 1, 0)(0, 1, 2, 1)$$

Considering the various pairs of vertices, we obtain the type sequence

$$[1(1-1), 5(1-2), 5(2-2), 1(1-3), 2(2-3)]$$

We now define the generation of a hadron. The generation for mesons is similar to the usual generations of the quark constituents except now we have a quark-antiquark pair so there are more categories. The generation $g(q_1\bar{q}_2)$ is defined by

$$g(u\bar{u}) = g(u\bar{s}) = 2, \quad g(s\bar{s}) = g(u\bar{c}) = 4, \quad g(s\bar{c}) = 6, \quad g(u\bar{b}) = 7, \quad g(c\bar{c}) = 9$$

$$g(s\bar{b}) = 10, \quad g(u\bar{t}) = 14, \quad g(c\bar{b}) = 17, \quad g(s\bar{t}) = 18, \quad g(c\bar{t}) = 20, \quad g(b\bar{b}) = 26$$

If a d, u' or d' appear instead of a u , the generation is the same as above. It will be useful later to introduce the *generation number*

$$\gamma(q_1\bar{q}_2) = 6g(q_1\bar{q}_2) + \frac{\widehat{s}(1 - \widehat{s})}{2} + \widehat{b}$$

where \widehat{s} and \widehat{b} are the number of strange and bottom quark constituents, respectively for $q_1\bar{q}_2$.

The generation for baryons follows a similar pattern and we only list some of the lighter ones

$$g(uud) = 3, \quad g(uus) = 4, \quad g(uss) = 5, \quad g(udc) = 6, \quad g(uuc) = g(sss) = 7$$

Again, the generation does not change if u or d is replaced by u' or d' . We also define the *generation number*

$$\gamma(q_1q_2q_3) = \begin{cases} 7g(q_1q_2q_3) & \text{if } n_{12} - n_{11} \leq 4 \\ 7g(q_1q_2q_3) + \frac{6}{4-\widehat{s}} + \widehat{c} & \text{otherwise} \end{cases}$$

The next two tables summarize the lighter hadrons.

Meson	Quark Form	γ	Configuration	Type Sequence
π^0	$u'\bar{u}'$	12	(1, 0, 1, 0)(1, 0, 1, 0)	[1(2 - 2)]
K^+	$u\bar{s}$	12	(0, 1, 1, 0)(1, 1, 2, 0)	[1(1 - 1), 3(1 - 2), 2(2, 2)]
ϕ^0	$s\bar{s}$	23	(1, 1, 2, 0)(1, 1, 2, 0)	[1(1 - 1), 4(1 - 2), 4(2 - 2)]
D^0	$u\bar{c}$	24	(0, 1, 1, 0)(0, 1, 2, 1)	[1(1 - 1), 3(1 - 2), 2(2 - 2), 1(1 - 3), 1(2 - 3)]
D_s^-	$s\bar{c}$	36	(1, 1, 2, 0)(0, 1, 2, 1)	[1(1 - 1), 4(1 - 2), 4(2 - 2), 1(1 - 3), 2(2 - 3)]
B^+	$u\bar{b}$	43	(0, 1, 1, 0)(1, 2, 2, 2)	[2(1 - 1), 4(1 - 2), 2(2 - 2), 2(1 - 3), 2(2 - 3)]
η_c^0	$c\bar{c}$	54	(0, 1, 2, 1)(0, 1, 2, 1)	[1(1 - 1), 4(1 - 2), 4(2 - 2), 2(1 - 3), 4(2 - 3)]
B_s^+	$s\bar{b}$	61	(1, 1, 2, 0)(1, 2, 2, 2)	[2(1 - 1), 6(1 - 2), 4(2 - 2), 2(1 - 3), 4(2 - 3)]

Table 3 (Meson Structures)

Baryon	Quark Form	γ	Configuration	Type Sequence
P^-	uud'	21	$(0, 1, 1, 0)(0, 1, 1, 0)(0, 1, 1, 0)$	$[3(1-1), 6(1-2), 3(2-2)]$
Λ^0	uds	28	$(0, 1, 1, 0)(1, 0, 1, 0)(1, 1, 2, 0)$	$[1(1-1), 5(1-2), 5(2, 2)]$
Σ^+	uus	30	$(0, 1, 1, 0)(0, 1, 1, 0)(1, 1, 2, 0)$	$[3(1-1), 8(1-2), 5(2-2)]$
Ξ^-	dss	38	$(1, 0, 1, 0)(1, 1, 2, 0)(1, 1, 2, 0)$	$[1(1-1), 6(1-2), 8(2-2)]$
Λ_c^+	udc	42	$(0, 1, 1, 0)(1, 0, 1, 0)(0, 1, 2, 1)$	$[1(1-1), 5(1-2), 5(2-2), 1(1-3), 2(2-3)]$
Σ_c^{++}	uuc	50	$(0, 1, 1, 0)(0, 1, 1, 0)(0, 1, 2, 1)$	$[3(1-1), 8(1-2), 5(2-2), 2(1-3), 2(2-3)]$
Ω^-	sss	55	$(1, 1, 2, 0)(1, 1, 2, 0)(1, 1, 2, 0)$	$[3(1-1), 12(1-2), 12(2-2)]$

Table 4 (Baryon Structures)

To apply (5.1) and (5.2) we choose interaction strengths from Table 2 according to the numbers given in the type sequence for the particular particle p . The resulting *interaction path* for such a choice becomes

$$s = (n_0(p), n_1(p), \dots, n_{10}(p)) \quad (5.5)$$

where n_0 is the number of 16s or 64s in type 1-1, n_1 is the number of 81s, n_2 is the number of 225s, ..., n_{10} is the number of 9216s in the order of Table 2. There are many possible interaction paths for each hadron. For example, K^+ has twelve possible interaction paths, some of which are

$$\begin{aligned} s_1 &= (1, 3, 0, 2, 0, \dots, 0) \\ s_2 &= (1, 3, 0, 0, 2, 0, \dots, 0) \\ s_3 &= (1, 3, 0, 1, 1, 0, \dots, 0) \end{aligned} \quad (5.6)$$

Notice that the integers in the interaction path must conform with the integers in the type sequence. For example, in s_3 comparing with (5.4) we have $1 + 1 = 2$.

The *interaction number* of an interaction path (5.5) for p is

$$n(s) = \sum_{j=0}^{10} j n_j(p)$$

An interaction path s for a hadron p is a *mass path* for p if $n(s) = \gamma(p)$. For example, the interaction numbers of the interaction paths in (5.6) are 9, 11, 10, respectively. Since $\gamma(K^+) = 12$, none of these are mass paths for K^+ .

If s given in (5.5) is a mass path for a meson $p = q_1\bar{q}_2$, then applying (5.1) the corresponding *mass prediction* is

$$m(s) = \frac{1}{2} [m(q_1) + m(q_2)] + (0.511) [16n_0(p) + 81n_1(p) + \cdots + 9216n_{10}(p)] \quad (5.7)$$

If $p = q_1q_2q_3$ is a baryon with mass path s given in (5.5), then applying (5.2) the corresponding mass prediction is

$$m(s) = \frac{1}{3} [m(q_1) + m(q_2) + m(q_3)] + (0.511) [64n_0(p) + 81n_1(p) + \cdots + 9216n_{10}(p)] \quad (5.8)$$

For consistency, we employ the quark masses derived in Section 4. The *mass* $m(p)$ of a hadron p is defined to be the average of the mass predictions for the mass paths of p . That is, if s_1, \dots, s_k are the mass paths for p then

$$m(p) = \frac{1}{k} \sum_{j=1}^k m(s_j) \quad (5.9)$$

In the exceptional case in which there does not exist a mass path for p we take $m(p) = m(s)$ where $n(s)$ is closest to $\gamma(p)$.

We now consider two examples. The simplest case is the π^0 meson which we propose has the quark composition

$$\pi^0 = u'\bar{u}' = (1, 0, 1, 0)(1, 0, 1, 0)$$

The generation number is $\gamma(\pi^0) = 12$. Since there is only a single type 2 – 2 interaction, the type sequence is $[1(2 - 2)]$. We then have only the two interaction paths

$$\begin{aligned} s_1 &= (0, 0, 0, 1, 0, \dots, 0) \\ s_2 &= (0, 0, 0, 0, 1, 0, \dots, 0) \end{aligned}$$

with $n(s_1) = 3$, $n(s_2) = 4$. As discussed earlier, since $n(s_2) = 4$ is closest to $\gamma(\pi^0) = 12$, applying (5.7) we obtain

$$m(\pi^0) = m(s_2) = 4.6 + (0.511)256 = 135.42(135)$$

In a similar way, we have that

$$\pi^+ = u'\bar{d} = (1, 0, 1, 0)(1, 0, 1, 0)$$

and

$$m(\pi^+) = 6.38 + (0.511)256 = 137.2(139.6)$$

We treat π^- in a similar way. In the sequel, when we have charged versions of the same particle, we usually only treat one.

Our second example is the meson $K^+ = u\bar{s}$ with type sequence (5.4) and generation number $\gamma(K^+) = 12$. Now there are 12 interaction paths given by

$$\begin{aligned} s_1 &= (1, 3, 0, 2, 0, \dots, 0), & s_2 &= (1, 3, 0, 0, 2, 0, \dots, 0) \\ s_3 &= (1, 3, 0, 1, 1, 0, \dots, 0), & s_4 &= (1, 2, 1, 2, 0, \dots, 0) \\ s_5 &= (1, 1, 2, 2, 0, \dots, 0), & s_6 &= (1, 2, 1, 0, 2, 0, \dots, 0) \\ s_7 &= (1, 1, 2, 0, 2, 0, \dots, 0), & s_8 &= (1, 2, 1, 1, 1, 0, \dots, 0) \\ s_9 &= (1, 1, 2, 1, 1, 0, \dots, 0), & s_{10} &= (1, 0, 3, 2, 0, \dots, 0) \\ s_{11} &= (1, 0, 3, 0, 2, \dots, 0), & s_{12} &= (1, 0, 3, 1, 1, 0, \dots, 0) \end{aligned}$$

The interaction numbers for these paths are

$$\begin{aligned} n(s_1) &= 9, & n(s_2) &= 11, & n(s_3) &= 10, & n(s_4) &= 10, & n(s_5) &= 11, & n(s_6) &= 12, \\ n(s_7) &= 13, & n(s_8) &= 11, & n(s_9) &= 12, & n(s_{10}) &= 12, & n(s_{11}) &= 14, & n(s_{12}) &= 13 \end{aligned}$$

We conclude that s_6 , s_9 and s_{10} are mass paths. Applying (5.7) these give the following mass predictions:

$$\begin{aligned} m(s_6) &= \frac{1}{2}(4.6 + 101.69) + (0.511)(16 + 2 \cdot 81 + 225 + 2 \cdot 256) = 520.71 \\ m(s_9) &= \frac{1}{2}(4.6 + 101.69) + (0.511)(16 + 81 + 2 \cdot 225 + 64 + 256) = 496.18 \\ m(s_{10}) &= \frac{1}{2}(4.6 + 101.69) + (0.511)(16 + 3 \cdot 225 + 2 \cdot 64) = 471.65 \end{aligned}$$

Applying (5.9) we obtain

$$m(K^+) = \frac{1}{3} [m(s_6) + m(s_9) + m(s_{10})] = 496.18(493.7)$$

6 Predicted Masses

We now compute masses for the hadrons listed in Tables 3 and 4. We have already found masses for π^0 and K^+ so we proceed for the others. All of these

predicted masses agree with experiment to within 6% and most of them agree much closer.

For the ϕ^0 meson we have $\gamma(\phi^0) = 23$ and mass paths

$$s_1 = (1, 0, 4, 1, 3, 0, \dots, 0), \quad s_2 = (1, 1, 3, 0, 4, 0, \dots, 0)$$

This gives the mass predictions

$$m(s_1) = 101.69 + (0.511)(16 + 4 \cdot 225 + 64 + 3 \cdot 256) = 994.92$$

$$m(s_2) = 101.69 + (0.511)(16 + 81 + 3 \cdot 225 + 4 \cdot 256) = 1019.45$$

The mass becomes

$$m(\phi^0) = \frac{1}{2} [m(s_1) + m(s_2)] = 1007.18(1020)$$

For the D^0 meson, we have $\gamma(D^0) = 24$ and the mass paths

$$\begin{aligned} s_1 &= (1, 0, 3, 2, 0, 1, 0, 1, 0, \dots, 0), & s_2 &= (1, 1, 2, 1, 1, 1, 0, 1, 0, \dots, 0) \\ s_3 &= (1, 1, 2, 2, 0, 0, 1, 1, 0, \dots, 0), & s_4 &= (1, 1, 2, 2, 0, 1, 0, 0, 1, 0, \dots, 0) \\ s_5 &= (1, 2, 1, 0, 2, 1, 0, 1, 0, \dots, 0), & s_6 &= (1, 2, 1, 1, 1, 0, 1, 1, 0, \dots, 0) \\ s_7 &= (1, 2, 1, 1, 1, 1, 0, 0, 1, 0, \dots, 0), & s_8 &= (1, 2, 1, 2, 0, 0, 1, 0, 1, 0, \dots, 0) \\ s_9 &= (1, 3, 0, 0, 2, 0, 1, 1, 0, \dots, 0), & s_{10} &= (1, 3, 0, 0, 2, 1, 0, 0, 1, 0, \dots, 0) \\ s_{11} &= (1, 3, 0, 1, 1, 0, 1, 0, 1, 0, \dots, 0) \end{aligned}$$

This gives the mass predictions

$$m(s_1) = 1414.69, \quad m(s_2) = 1439.22, \quad m(s_3) = 1602.74$$

$$m(s_4) = 2056.5, \quad m(s_5) = 1463.74, \quad m(s_6) = 1627.26$$

$$m(s_7) = 2081.03, \quad m(s_8) = 2244.55, \quad m(s_9) = 1651.79$$

$$m(s_{10}) = 2105.56, \quad m(s_{11}) = 2269.08$$

The mass becomes

$$m(D^0) = \frac{1}{11} [m(s_1) + \dots + m(s_{11})] = 1814.2(1864.61)$$

For the D_s^- meson, we have $\gamma(D_s^-) = 36$ and the mass paths

$$s_1 = (1, 3, 1, 4, 0, 1, 0, 2, 0, \dots, 0), \quad s_2 = (1, 4, 0, 3, 1, 1, 0, 2, 0, \dots, 0)$$

$$s_3 = (1, 4, 0, 4, 0, 0, 1, 2, 0, \dots, 0), \quad s_4 = (1, 4, 0, 4, 0, 1, 0, 1, 1, 0, \dots, 0)$$

This gives the mass predictions

$$m(s_1) = 1742.25, \quad m(s_2) = 1766.78, \quad m(s_3) = 1930.3, \quad m(s_4) = 2499.05$$

The mass becomes

$$m(D_s^-) = \frac{1}{4} [m(s_1) + m(s_2) + m(s_3) + m(s_4)] = 1984.6(1969)$$

For the B^+ meson, we have $\gamma(B^+) = 43$ and the mass paths

$$\begin{aligned} s_1 &= (2, 0, 4, 0, 2, 0, 2, 1, 1, 0, \dots, 0), & s_2 &= (2, 1, 3, 0, 2, 0, 2, 0, 2, 0, \dots, 0) \\ s_3 &= (2, 0, 4, 1, 1, 0, 2, 0, 2, 0, \dots, 0), & s_4 &= (2, 0, 4, 0, 2, 1, 1, 0, 2, 0, \dots, 0) \end{aligned}$$

This gives the mass predictions

$$m(s_1) = 4825.38, \quad m(s_2) = 5467.19, \quad m(s_3) = 5442.65, \quad m(s_4) = 5279.13$$

The mass becomes

$$m(B^+) = \frac{1}{4} [m(s_1) + m(s_2) + m(s_3) + m(s_4)] = 5253.59(5279)$$

For the η_c^0 meson, we have $\gamma(\eta_c^0) = 54$ and there is only one mass path given by

$$s = (1, 4, 0, 4, 0, 2, 0, 4, 0, \dots, 0)$$

This gives the mass

$$\begin{aligned} m(\eta_c^0) = m(s) &= 1283.63 + (0.511)(16 + 4 \cdot 81 + 4 \cdot 64 + 2 \cdot 64 + 4 \cdot 625) \\ &= 2931.09(2983.6) \end{aligned}$$

For the B_s^+ meson, we have $\gamma(B_s^+) = 61$. There are 35 mass paths which is too many to write down so we display a few:

$$\begin{aligned} s_1 &= (2, 1, 5, 4, 0, 2, 0, 4, 0, \dots, 0), & s_2 &= (2, 2, 4, 3, 1, 2, 0, 4, 0, \dots, 0) \\ s_{34} &= (2, 6, 0, 3, 1, 2, 0, 0, 4, 0, \dots, 0), & s_{35} &= (2, 6, 0, 4, 0, 1, 1, 0, 4, 0, \dots, 0) \end{aligned}$$

In summary, the mass becomes

$$m(B_s^+) = \frac{1}{35} [m(s_1) + \dots + m(s_{35})] = 5366.53(5370)$$

We now consider baryons. The proton P^+ has $\gamma(P^+) = 21$ and mass paths

$$\begin{aligned} s_1 &= (3, 0, 6, 3, 0, \dots, 0), & s_2 &= (3, 1, 5, 2, 1, 0, \dots, 0) \\ s_3 &= (3, 2, 4, 1, 2, 0, \dots, 0), & s_4 &= (3, 3, 3, 0, 3, 0, \dots, 0) \end{aligned}$$

This gives the mass predictions

$$\begin{aligned} m(s_1) &= \frac{1}{3}(4.6 + 4.6 + 8.17) + (0.511)(3 \cdot 64 + 6 \cdot 225 + 3 \cdot 64) = 891.86 \\ m(s_2) &= \frac{1}{3}(4.6 + 4.6 + 8.17) + (0.511)(3 \cdot 64 + 81 + 5 \cdot 225 + 2 \cdot 64 + 256) \\ &= 916.39 \\ m(s_3) &= \frac{1}{3}(4.6 + 4.6 + 8.17) + (0.511)(3 \cdot 64 + 2 \cdot 81 + 4 \cdot 225 + 64 + 2 \cdot 256) \\ &= 940.92 \\ m(s_4) &= \frac{1}{3}(4.6 + 4.6 + 8.17) + (0.511)(3 \cdot 64 + 3 \cdot 81 + 3 \cdot 225 + 3 \cdot 256) \\ &= 965.45 \end{aligned}$$

and mass

$$m(P^+) = \frac{1}{4}[m(s_1) + \dots + m(s_4)] = 928.66(938.27)$$

The neutron $N^0 = ud'd'$ is similar with mass

$$m(N^0) = 929.75(939.57)$$

The Λ^0 baryon has $\gamma(\Lambda^0) = 28$ and mass paths

$$s_1 = (1, 0, 5, 2, 3, 0, \dots, 0), \quad s_2 = (1, 1, 4, 1, 4, 0, \dots, 0), \quad s_3 = (1, 2, 3, 0, 5, 0, \dots, 0)$$

This gives the mass predictions

$$m(s_1) = 1103.59, \quad m(s_2) = 1128.12, \quad m(s_3) = 1152.64$$

and mass

$$m(\Lambda^0) = 1128.12(1115.6)$$

The Σ^+ baryon has $\gamma(\Sigma^+) = 30$ and mass paths

$$s_1 = (3, 1, 7, 5, 0, \dots, 0), \quad s_2 = (3, 2, 6, 4, 1, 0, \dots, 0), \quad s_3 = (3, 3, 5, 3, 2, 0, \dots, 0)$$

$$s_4 = (3, 4, 4, 2, 3, 0, \dots, 0), \quad s_5 = (3, 5, 3, 1, 4, 0, \dots, 0), \quad s_6 = (3, 6, 2, 0, 5, 0, \dots, 0)$$

This gives the mass predictions

$$\begin{aligned} m(s_1) &= 1144.81, \quad m(s_2) = 1169.34, \quad m(s_3) = 1193.87 \\ m(s_4) &= 1218.40, \quad m(s_5) = 1242.29, \quad m(s_6) = 1267.45 \end{aligned}$$

and mass

$$m(\Sigma^+) = 1206.13(1189.4)$$

The Ξ^- baryon has $\gamma(\Xi^-) = 38$ and mass paths

$$\begin{aligned} s_1 &= (1, 0, 6, 6, 2, 0, \dots, 0), \quad s_2 = (1, 1, 5, 5, 3, 0, \dots, 0), \quad s_3 = (1, 2, 4, 4, 4, 0, \dots, 0) \\ s_4 &= (1, 3, 3, 3, 5, 0, \dots, 0), \quad s_5 = (1, 4, 2, 2, 6, 0, \dots, 0), \quad s_6 = (1, 5, 1, 1, 7, 0, \dots, 0) \\ s_7 &= (1, 6, 0, 0, 8, 0, \dots, 0) \end{aligned}$$

We then have the mass predictions

$$\begin{aligned} m(s_1) &= 1250.93, \quad m(s_2) = 1274.50, \quad m(s_3) = 1299.98, \quad m(s_4) = 1324.51 \\ m(s_5) &= 1349.04, \quad m(s_6) = 1373.57, \quad m(s_7) = 1398.09 \end{aligned}$$

and mass

$$m(\Xi^-) = 1324.51(1321.3)$$

The Λ_c^+ baryon has $\gamma(\Lambda_c^+) = 42$ and mass paths

$$\begin{aligned} s_1 &= (1, 2, 3, 5, 0, 1, 0, 2, 0, \dots, 0), & s_2 &= (1, 3, 2, 4, 1, 1, 0, 2, 0, \dots, 0) \\ s_3 &= (1, 3, 2, 5, 0, 0, 1, 2, 0, \dots, 0), & s_4 &= (1, 3, 2, 5, 0, 1, 0, 1, 1, 0, \dots, 0) \\ s_5 &= (1, 4, 1, 3, 2, 1, 0, 2, 0, \dots, 0), & s_6 &= (1, 4, 1, 5, 0, 1, 0, 0, 2, 0, \dots, 0) \\ s_7 &= (1, 4, 1, 4, 1, 0, 1, 2, 0, \dots, 0), & s_8 &= (1, 4, 1, 4, 1, 1, 0, 1, 1, 0, \dots, 0) \\ s_9 &= (1, 4, 1, 5, 0, 0, 1, 1, 1, 0, \dots, 0), & s_{10} &= (1, 5, 0, 2, 3, 1, 0, 2, 0, \dots, 0) \\ s_{11} &= (1, 5, 0, 5, 0, 0, 1, 0, 2, 0, \dots, 0), & s_{12} &= (1, 5, 0, 4, 1, 1, 0, 0, 2, 0, \dots, 0) \\ s_{13} &= (1, 5, 0, 4, 1, 0, 1, 1, 1, 0, \dots, 0), & s_{14} &= (1, 5, 0, 3, 2, 0, 1, 2, 0, 0, \dots, 0) \\ s_{15} &= (1, 5, 0, 3, 2, 1, 0, 1, 1, 0, \dots, 0) \end{aligned}$$

This gives the mass predictions

$$\begin{aligned}
m(s_1) &= 1727.52, & m(s_2) &= 1752.05, & m(s_3) &= 1915.57, & m(s_4) &= 2369.33 \\
m(s_5) &= 1776.57, & m(s_6) &= 3011.15, & m(s_7) &= 1940.09, & m(s_8) &= 2393.86 \\
m(s_9) &= 2081.03, & m(s_{10}) &= 2244.55, & m(s_{11}) &= 1651.79, & m(s_{12}) &= 1602.74 \\
m(s_{13}) &= 2557.38, & m(s_{14}) &= 1801.10, & m(s_{15}) &= 3199.20, & &= 3035.68 \\
m(s_{16}) &= 2581.91, & m(s_{17}) &= 1964.62, & m(s_{18}) &= 2418.39 & &
\end{aligned}$$

The mass becomes

$$m(\Lambda_c^+) = \frac{1}{15} [m(s_1) + \dots + m(s_{15})] = 2296.29(2286)$$

The Ω^- baryon has $\gamma(\Omega^-) = 55$ and mass paths

$$\begin{aligned}
s_1 &= (3, 5, 7, 12, 0, \dots, 0), & s_2 &= (3, 6, 6, 11, 1, 0, \dots, 0), & s_3 &= (3, 7, 5, 10, 2, 0, \dots, 0) \\
s_4 &= (3, 8, 4, 9, 3, 0, \dots, 0), & s_5 &= (3, 9, 3, 8, 4, 0, \dots, 0), & s_6 &= (3, 10, 2, 7, 5, 0, \dots, 0) \\
s_7 &= (3, 11, 1, 6, 6, 0, \dots, 0), & s_8 &= (3, 12, 0, 5, 7, 0, \dots, 0)
\end{aligned}$$

We then have the mass predictions

$$\begin{aligned}
m(s_1) &= 1604.03, & m(s_2) &= 1628.56, & m(s_3) &= 1653.09, & m(s_4) &= 1677.61, \\
m(s_5) &= 1702.14, & m(s_6) &= 1726.67, & m(s_7) &= 1751.20, & m(s_8) &= 1775.73
\end{aligned}$$

The mass becomes

$$m(\Omega^-) = \frac{1}{8} [m(s_1) + \dots + m(s_8)] = 1689.88(1672.5)$$

Our last example is the Σ_c^{++} baryon with $\gamma(\Sigma_c^{++}) = 50$ and mass paths

$$\begin{aligned}
s_1 &= (3, 5, 3, 5, 0, 2, 0, 2, 0, \dots, 0), & s_2 &= (3, 6, 2, 4, 1, 2, 0, 2, 0, \dots, 0) \\
s_3 &= (3, 6, 2, 5, 0, 1, 1, 2, 0, \dots, 0), & s_4 &= (3, 6, 2, 5, 0, 2, 0, 1, 1, 0, \dots, 0) \\
s_5 &= (3, 7, 1, 3, 2, 2, 0, 2, 0, \dots, 0), & s_6 &= (3, 7, 1, 5, 0, 0, 2, 2, 0, \dots, 0) \\
s_7 &= (3, 7, 1, 5, 0, 2, 0, 0, 2, \dots, 0), & s_8 &= (3, 7, 1, 4, 1, 1, 1, 2, 0, \dots, 0) \\
s_9 &= (3, 7, 1, 4, 1, 2, 0, 1, 1, 0, \dots, 0), & s_{10} &= (3, 7, 1, 5, 0, 1, 1, 1, 1, 0, \dots, 0) \\
s_{11} &= (3, 8, 0, 2, 3, 2, 0, 2, 0, \dots, 0), & s_{12} &= (3, 8, 0, 3, 2, 1, 1, 2, 0, \dots, 0) \\
s_{13} &= (3, 8, 0, 4, 1, 0, 2, 2, 0, \dots, 0), & s_{14} &= (3, 8, 0, 3, 2, 2, 0, 1, 1, 0, \dots, 0) \\
s_{15} &= (3, 8, 0, 4, 1, 2, 0, 0, 2, 0, \dots, 0), & s_{16} &= (3, 8, 0, 5, 0, 0, 2, 1, 1, 0, \dots, 0)
\end{aligned}$$

$$s_{17} = (3, 8, 0, 5, 0, 1, 1, 0, 2, 0, \dots, 0), \quad s_{18} = (3, 8, 0, 4, 1, 1, 1, 1, 1, 0, \dots, 0)$$

This gives the mass predictions

$$\begin{aligned} m(s_1) &= 1948.61, & m(s_2) &= 1973.41, & m(s_3) &= 2136.66, & m(s_4) &= 2590.43 \\ m(s_5) &= 1997.67, & m(s_6) &= 2324.71, & m(s_7) &= 3232.25, & m(s_8) &= 2161.19 \\ m(s_9) &= 2614.96, & m(s_{10}) &= 2778.48, & m(s_{11}) &= 2022.20, & m(s_{12}) &= 2185.72 \\ m(s_{13}) &= 2349.24, & m(s_{14}) &= 2639.49, & m(s_{15}) &= 3256.77, & m(s_{16}) &= 2966.53 \\ m(s_{17}) &= 3420.29, & m(s_{18}) &= 2803.01 \end{aligned}$$

The mass becomes

$$m(\Sigma_c^{++}) = \frac{1}{18} [m(s_1) + \dots + m(s_{18})] = 2522.31(2452)$$

We close with two remarks. First, we have not considered electric forces in this work. The strengths that we introduced evidently correspond to the weak and strong nuclear forces. Presumably, the neglect of electric forces is responsible for the small discrepancies in our mass computations. Second, we have only presented basic hadrons and have not considered excited states or resonances. For example

$$\omega^0 = u\bar{u} = (0, 1, 1, 0, 0, 1, 1, 0)$$

is a resonances with interaction sequence

$$[1(1-1), 2(1-2), 1(2-2)]$$

we have that $\gamma(\omega^0) = 12$ and the interaction path

$$s = (1, 0, 2, 0, 1, 0, \dots, 0)$$

has interaction number $n(s) = 8$ which is the closest to $\gamma(\omega^0)$. The usual mass prediction $m(\omega^0) = m(s)$ is much smaller than the experimental value. However, if we employ other interaction strengths from Table 1 we can obtain

$$m(\omega^0) = 4.6 + (0.511)(64 + 2 \cdot 225 + 1024) = 790.5(782)$$

This indicates that more flexibility is required if resonances are to be included.

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