Submit your typed answers to the questions below using the department's Gitlab server by September 29, 2015 @ 11:59pm. Put your PDF into a folder named `theory_assignment1`.

1. Let p, q, and r be the propositions:

   - p: You get an A on the final exam.
   - q: You do every exercise in the textbook.
   - r: You get an A in this class

   Write these propositions using p, q, and r and logical connectives (including negations).

   (a) You get an A in this class, but you do not do every exercise in this book.

   (b) You get an A on the final, you do every exercise in the textbook, and you get an A in this class.

   (c) To get an A in this class, it is necessary for you to get an A on the final.

   (d) You get an A on the final, but you don't do every exercise in the textbook; nevertheless, you get an A in this class.

   (e) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

2. Construct a truth table for each of these compound propositions

   (a) $(p \vee q) \implies (p \oplus q)$

   (b) $p \implies \neg q$

   (c) $(\neg p \iff \neg q) \iff (\neg q \vee r)$

   (d) $((p \implies q) \implies r) \implies s$

   (e) $(p \implies q) \oplus (\neg p \iff q)$

3. Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations (i.e. your terms will be $P(0)$, $P(1)$, etc.).

   (a) $\exists x,\ P(x)$

   (b) $\forall x,\ P(x)$

   (c) $\exists x,\ \neg P(x)$

   (d) $\forall x,\ \neg P(x)$

   (e) $\neg \forall x,\ P(x)$

4. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

   (a) Something is not in the correct place.

   (b) Everything is in the correct place and in excellent condition.

   (c) All tools are in the correct place and are in excellent condition.

(d) Nothing is in the correct place and is in excellent condition.

(e) Exactly one of your tools is not in the correct place, but it is in excellent condition.

5. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifiers or an expression involving logical connectives).

   (a) $\neg\exists y \exists x,\ P(x, y)$
   (b) $\neg\forall x \exists y,\ P(x, y)$
   (c) $\neg\exists y\,(Q(y) \wedge \forall x,\ \neg R(x, y))$
   (d) $\neg\exists y\,(\exists x,\ R(x, y) \vee \forall x,\ S(x, y))$
   (e) $\neg\exists y\,(\forall x \exists z,\ T(x, y, z)\ \vee\ \exists x \forall z U(x, y, z))$

6. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

   (a) $A \bigcup B$
   (b) $A \bigcap B$
   (c) $A - B$
   (d) $B - A$

7. Using set membership tables prove the following two forms of DeMorgan's law.

   (a) $\overline{A \bigcup B} = \overline{A} \bigcap \overline{B}$
   (b) $\overline{A \bigcap B} = \overline{A} \bigcup \overline{B}$

8. Using set membership tables, prove $A \oplus B = (A \bigcup B) - (A \bigcap B)$