Submit your typed answers to the questions below using the department's Gitlab server by October 16, 2015 @ 11:59pm. Put your PDF into a folder named `theory_assignment3`.

Let $\vec{x} = \langle x_{w-1}, x_{w-2}, \ldots, x_0 \rangle$ be a binary vector of length w. Let $B2U\colon 2^w \to \mathbb{N}$ be the function that maps from binary vectors of length w to the natural numbers using an unsigned encoding. Let $B2T\colon 2^w \to \mathbb{Z}$ be the function that maps from binary vectors of length w to the integers using a two's complement encoding.

Then

$$B2U(\vec{x}) = \sum_{i=0}^{w-1} x_i 2^i \qquad \text{and} \qquad B2T(\vec{x}) = -x_{w-1}2^{w-1} + \sum_{i=0}^{w-2} x_i 2^i.$$

1. Provide an algorithm to add two unsigned encoded numbers together and prove its correctness. *Hint:* Carry bits

2. Provide an algorithm to add two two's complement encoded numbers together and prove its correctness. *Hint:* The algorithm should be the same as for unsigned encoded numbers.

3. Prove that division in unsigned numbers is equivalent to multiplication. *Hint:* Show that division by x is equivalent to multiplication by another number x'. Finding such an x' is related to the Euclidean Algorithm.