

## ERRATA TO “GAMES AND GENERAL DISTRIBUTIVE LAWS IN BOOLEAN ALGEBRAS”

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We are grateful to B. Balcar for pointing out the following error in [1]: In Example 2 (and hence Example 1), if  $\eta > \omega$ , then  $\eta$  regular and  $\diamond_{\eta^+}$  do not suffice to construct an  $\eta^+$ -Suslin algebra. The original construction breaks down at the point where we “choose” the appropriate branches  $B_t$ , since  $\diamond_{\eta^+}$  is not strong enough to guarantee their existence. Balcar suggested the following modified construction, assuming  $\eta^{<\eta} = \eta$  and  $\diamond_{\eta^+}(E(\eta))$ .

**Replacement for Example 2.** *If  $\eta^{<\eta} = \eta$ ,  $\lambda \leq \min(\kappa, \eta)$ , and  $\diamond_{\eta^+}(E(\eta))$ , then there is an  $\eta^+$ -Suslin algebra in which  $\mathcal{G}_{<\lambda}^\eta(\kappa)$  is undetermined.*

Recall that  $E(\eta)$  denotes  $\{\alpha < \eta^+ : \text{cf}(\alpha) = \eta\}$ . Fix a  $\diamond_{\eta^+}(E(\eta))$ -sequence; i.e. a sequence  $\langle A_\alpha : \alpha \in E(\eta) \rangle$  such that  $\forall \alpha \in E(\eta)$ ,  $A_\alpha \subseteq \alpha$  and  $\forall A \subseteq \eta^+$ ,  $\{\alpha \in E(\eta) : A \cap \alpha = A_\alpha\}$  is stationary. Let (d) be the statement: “ $\forall \beta < \alpha$ , if  $\text{cf}(\beta) < \eta$ , then all  $\beta$ -branches of  $T_\beta$  extend to  $\text{Lev}(\beta)$ ; if  $\text{cf}(\beta) = \eta$ , then for each  $t \in T_\beta$  there is a  $(\beta + 1)$ -branch in  $T_{\beta+1}$  containing  $t$ .” We construct  $T$  so that (d) holds for each  $\alpha < \eta^+$ .

Suppose  $T_\alpha$  has been constructed and (d) holds for  $\alpha$ . If  $\alpha$  is not a limit ordinal, proceed as before. If  $\omega \leq \text{cf}(\alpha) < \eta$ , then there are at most  $\eta$ -many  $\alpha$ -branches through  $T_\alpha$ , since  $\eta^{<\eta} = \eta$ . Extend each of them to  $\text{Lev}(\alpha)$ . If  $\text{cf}(\alpha) = \eta$ , consider statements (a)-(c) in the combinations as before. Since (d) holds for  $\alpha$ , the appropriate  $\alpha$ -branches  $B_t$  now exist in  $T_\alpha$ . Extend the branches  $B_t$  to  $\text{Lev}(\alpha)$ .

In the argument that  $\mathcal{G}_{<\lambda}^\eta(\kappa)$  is undetermined in r.o.  $(T^*)$  we should have bijected  $\alpha$  with  $\eta$  so that P1 will play the partitions  $\mathcal{P}_\beta$  ( $\beta < \alpha$ ) in a game of length  $\eta$ . The proofs that  $T$  is an  $\eta^+$ -Suslin tree and  $\mathcal{G}_{<\lambda}^\eta(\kappa)$  is undetermined in r.o.  $(T^*)$  proceed as before, using  $\langle A_\alpha : \alpha \in E(\eta) \rangle$  in place of the former  $\diamond_{\eta^+}$ -sequence.

The “ $\eta$ ” in the first sentence of the paragraph on the construction of  $\text{Lev}(\alpha + 1)$  should be an “ $\eta^+$ ”, so that it reads “Let  $\alpha < \eta^+$  and suppose  $\text{Lev}(\alpha)$  and  $\mathcal{P}_\alpha$  have been constructed.”

In the first paragraph of the Introduction in [1], a result of Foreman was not stated in its full strength. Foreman showed that (in our notation) for each cardinal  $\eta$  (not just successor cardinals), the  $(\eta, \infty)$ -d.l. is equivalent to P1 not having a winning strategy in the game played like  $G(\mathbf{P}, \eta)$  except that P2 chooses first at limit ordinals [2].

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## REFERENCES

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2. M. Foreman, *Games played on Boolean algebras*, J. Symbolic Logic **48** (3) (1983), 714-723.  
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