Sample Midterm Exam

Math 112Z
9/28/08

Name: ______________________________

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the poin.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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1. (*20 points*) Determine whether the following sequences are convergent or divergent. If they converge, find their limit.

a. (*6 pts*)

$$\{a_n\} = 1, -1, 1/2, -1/2, 1/3, -1/3, ...$$

b. (*7 pts*)

$$b_n = n^2 e^{-n}$$

c. (*7 pts*)

$$c_n = \sum_{j=1}^{n} \frac{1}{j}.$$

2. (*20 points*) Determine whether the following series converge or diverge. If they converge, find their value. If they diverge, indicate how you know.

a. (*10 pts*)

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}.$$

b. (*10 pts*)

$$\sum_{k=1}^{\infty} \sin^2(k).$$

3. (*20 points*) Use the integral test to determine whether or not

$$\sum_{i=1}^{\infty} \frac{1}{x^2 + 6x + 10}.$$

is convergent or divergent. *HINT: Complete the square.*

4. (*20 points*) For each statement, mark it true of false. If it is false give a (counter)example. If it is true give a reason - if the reason is a theorem, state the theorem, otherwise give a brief proof. No credit for answers without a correct reason or example.

a. (*3 pts*) If $\sum_{i=1}^{\infty} a_i$ converges, then the sequence $\{a_i\}$ is also convergent.

b. (*3 pts*)

$$\int_1^{xy} \frac{1}{t} dt = \int_1^{x} \frac{1}{t} dt + \int_1^{y} \frac{1}{t} dt.$$

c. (*3 pts*) If $\{a_i\}$ is a bounded, increasing sequence whose terms are positive (so $a_i > 0$ for every i) then $\sum_{i=1}^{\infty} a_i$ converges.

d. (*4 pts*) If $\{b_i\}$ is a decreasing sequence then $\{b_i\}$ diverges.

e. (*4 pts*) If $\sum_{j=1}^{n} a_j$ is divergent, then $\lim_{n\to\infty} a_j \neq 0$.

f. (*3 pts*) The exponential function $\exp(x) = e^x$ is the only function whose derivative is itself.

5. (*20 points*)

a. (*10 pts*) Define convergent series. Give one example of a convergent series, and one example of a divergent series. (Make sure your examples are clearly labeled as to which is which.

b. (*10 pts*) For what values of c does the series $\sum_{n=1}^{\infty} \frac{c}{n^c}$? converge?

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