Sample Midterm Exam

Math 112Z
9/28/08

Name: _______________________

Read all of the following information before starting the exam:

• READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!

• Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

• A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.

• Circle or otherwise indicate your final answers.

• Please keep your written answers clear, concise and to the point.

• This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!

• Turn off cellphones, etc.

• Good luck!

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2
3
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∑
1. (20 points) Determine whether the following series converge absolutely, converge conditionally or diverge.
   a. (10 pts)
   \[
   \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^{3/2} + 1}.
   \]
   
   b. (10 pts)
   \[
   \sum_{n=1}^{\infty} \frac{(n!)^2}{((2n)!)^2}.
   \]
2. (20 points) Determine the radius and interval of convergence for the following power series.

a. (10 pts)
\[ \sum_{n=1}^{\infty} (\ln n)^n x^n \]

b. (10 pts)
\[ \sum_{n=0}^{\infty} \left( \frac{2n + 3}{n + 2} \right)^n x^{n^2} \]
3. (20 points) Consider the power series:

\[ f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{ne^n} \]

a. (10 pts) Find the radius and interval of convergence for the power series.

b. (5 pts) Find a power series representation for \( f'(x) \). What is the interval and radius of convergence for this new power series?

c. (5 pts) Find a power series representation for \( f(x^2) \). What is the interval and radius of convergence for this new power series?
4. (20 points) Give an example of each of the following:

a. (5 pts) A power series with interval of convergence $(0, 2]$.

b. (5 pts) A power series with radius of convergence $R = \infty$.

c. (5 pts) A series which is absolutely convergence, but is not alternating or strictly positive.

d. (5 pts) Two series such that $f_n < g_n$ for all $n$, $\sum_{n=0}^{\infty} f_n$ diverges and $\sum_{n=0}^{\infty} g_n$ converges. (Hint: what hypothesis of the comparison test is missing?)
5. (20 points)

a. (10 pts) Use the power series for \( \ln(1 - x) \) to find a power series for \( \ln(x) \). What is the radius and interval of convergence for this power series?

b. (10 pts) Note that \( e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \). How many terms must be used to estimate \( e^{-1} \) with an error of at most \( \frac{1}{120} \)? (If you cannot solve explicitly for \( n \), just leave an expression, but it is set up to have a nice answer.)