Sample Midterm Exam

Math 112Z   **Name:** _____

9/28/08

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.

- Circle or otherwise indicate your final answers.

- Please keep your written answers clear, concise and to the point.

- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!

- Turn off cellphones, etc.

- Good luck!

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$\sum$	

1. (*20 points*)   **a.** (*10 pts*) Use Taylor's Theorem to find a Taylor Series for the function $\ln(x)$ centered at e.

b. (*10 pts*)   Use your series to find the limit:

$$\lim_{x \to e} \frac{1 - \ln(x)}{x - e}.$$

2. (*20 points*)

   a. (*10 pts*)   Use the Maclaurin series for $\sin(x)$ to write an infinite series with value $\int_0^{.1} \sin(x^2)dx$.

   b. (*10 pts*)   Suppose you use just the first term of your series as an approximation for $\int_0^{.1} \sin(x^2)$. Give a bound on how far off you are from the actual value.

3. (*20 points*) Using the Maclaurin series for $\cos(x)$, find the first 3 terms of the Maclaurin series for $\sec(x)$.

Hint: $\sec(x) = \frac{1}{\cos(x)}$ - *divide!*.

4. (*20 points*)   **a.** (*10 pts*)   Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}.$$

b. (*10 pts*)   Explain why $1 - x < e^{-x}$ for all numbers x.
Hint: Use the Taylor series for e^x. The statement is automatically true if $x > 1$.

5. (*20 points*)
a. (*10 pts*)   Find

$$\int 2x \arctan(x)dx$$

Hint: The fact that $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$ may come in handy.

b. (*10 pts*)   Find

$$\int \frac{4}{(x + 1)(x - 1)^2}.$$