Sample Midterm Exam

Math 112Z
9/28/08

Name: ____________________________

Read all of the following information before starting the exam:

• READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!

• Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

• A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.

• Circle or otherwise indicate your final answers.

• Please keep your written answers clear, concise and to the point.

• This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!

• Turn off cellphones, etc.

• Good luck!

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2
3
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∑
1. (20 points)  
a. (10 pts) Use Taylor’s Theorem to find a Taylor Series for the function \( \ln(x) \) centered at \( e \).

b. (10 pts) Use your series to find the limit:

\[
\lim_{x \to e} \frac{1 - \ln(x)}{x - e}.
\]
2. (20 points)
   a. (10 pts) Use the Maclaurin series for \( \sin(x) \) to write an infinite series with value 
   \( \int_{0}^{1} \sin(x^2) \, dx \).

   b. (10 pts) Suppose you use just the first term of your series as an approximation for 
   \( \int_{0}^{1} \sin(x^2) \, dx \). Give a bound on how far off you are from the actual value.
3. \textit{(20 points)} Using the Maclaurin series for $\cos(x)$, find the first 3 terms of the Maclaurin series for $\sec(x)$. \\
\textit{Hint:} $\sec(x) = \frac{1}{\cos(x)}$ - divide!
4. (20 points) a. (10 pts) Find the sum of the following series:

\[ \sum_{n=0}^{\infty} \frac{3^n}{5^n n!}. \]

b. (10 pts) Explain why \(1 - x < e^{-x}\) for all numbers \(x\).

*Hint:* Use the Taylor series for \(e^x\). The statement is automatically true if \(x > 1\).
5. (20 points)

a. (10 pts) Find \[ \int 2x \arctan(x) \, dx \]

*Hint:* The fact that \[ \frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1} \] may come in handy.

b. (10 pts) Find \[ \int \frac{4}{(x + 1)(x - 1)^2} \, dx. \]