Sample Midterm Exam
Math 112Z
9/28/08

Read all of the following information before starting the exam:

• READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!

• Show all work, clearly and in order, if you want to get full credit. I reserve the right to
take off points if I cannot see how you arrived at your answer (even if your final answer is
correct).

• A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.

• Circle or otherwise indicate your final answers.

• Please keep your written answers clear, concise and to the point.

• This test has xxx problems and is worth xxx points. It is your responsibility to make sure
that you have all of the pages!

• Turn off cellphones, etc.

• Good luck!

<table>
<thead>
<tr>
<th>1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>∑</td>
<td></td>
</tr>
</tbody>
</table>
1. (20 points)  

a. (10 pts)  One way of expressing the function $\sinh(x)$ is

$$\sinh(x) = \frac{e^x - e^{-x}}{2}.$$  

Use this and the Maclaurin series for $e^x$ to find a Maclaurin series for $\sinh(x)$.

b. (10 pts)  If $f(x) = \sinh(x)$ find a series representation for $f'(x^2)$. 
2. (20 points)
   a. (10 pts) Suppose $f(x)$ is an increasing concave up function at $x = 0$. Is it possible that
   \[ f(x) = 3 + 2x - x^2 + .3x^4 + 0.02x^5 + \ldots. \]
   Why or why not?

   b. (10 pts) Suppose $f(x) = \sum_{n=1}^{\infty} \frac{1}{n} x^n$. What is $f^{10}(0)$?
3. \((20 \text{ points})\) Using the Maclaurin series for \(\cos(x)\), and \(e^x\) find the first three non-zero terms of the Maclaurin series for \(\cos(x)e^x\).
4. (20 points) a. (10 pts) Suppose the Maclaurin series for $f(x)$ starts:

$$f(x) = 1 - \frac{1}{2}x - \frac{1}{10}x^2 + \ldots$$

and we know that $f^{(3)}(x) < \frac{1}{100}$ for $|x| < 1$. Use Taylor’s inequality to bound the error in the statement

$$f(0.1) \approx 1 - \frac{1}{2}(0.1) + \frac{1}{10}(0.1)^2.$$ 

b. (10 pts) Use series representations for $\cos(x)$, $\sin(x)$ to find

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x \sin(x)}$$
5. (20 points)
a. (10 pts) Find \[ \int \arcsin(x) \, dx \]

*Hint:* Try integration by parts.

b. (10 pts) Find \[ \int \frac{4}{(x - 1)(x^2 + 1)} \, dx. \]