1. (1.2.9) If \( C_1, C_2, C_3, \ldots \) are sets such that \( C_k \supseteq C_{k+1}, k = 1, 2, 3, \ldots, \infty, \) we define \( \lim_{k \to \infty} C_k \) as the intersection \( \bigcap_{k=1}^{\infty} C_k = C_1 \cap C_2 \cap \ldots. \) Find \( \lim_{k \to \infty} C_k \) for the following, and draw a picture of a typical \( 'C_k' \) on the line or plane, as appropriate:

a. \( C_k = \{ x : 2 - 1/k < x \leq 2 \}, k = 1, 2, 3, \ldots. \)

b. \( C_k = \{ x : 2 < x \leq 2 + \frac{1}{k} \}, k = 1, 2, 3, \ldots. \)

c. \( C_k = \{ (x, y) : 0 \leq x^2 + y^2 \leq \frac{1}{k} \}, k = 1, 2, 3, \ldots. \)

**Note:** In addition to the book problem, I ask for a picture of the set.

2. (1.2.4) Let \( \Omega \) denote the set of points interior to or on the boundary of a cube with edge of length 1. Moreover, say the cube is in the first octant with one vertex at the point \((0, 0, 0)\) and an opposite vertex at the point \((1, 1, 1)\). Let \( Q(C) = \int \int \int_C dx\,dy\,dz. \)

(a.) If \( C \subseteq \Omega \) is the set \( \{ (x, y, z) : 0 < x < y < z < 1 \} \) compute \( Q(C) \). Describe the set \( C \) in words (or picture).

(b.) If \( C \subseteq \Omega \) is the set \( \{ (x, y, z) : 0 < x = y = z < 1 \} \) compute \( Q(C) \). Describe the set \( C \) in words (or picture).

**Note:** In addition to the book problem, I ask for a description of the set.

3. (1.3.6) Suppose \( \Omega = \mathbb{R} \). For \( C \subseteq \Omega \) such that \( \int_C e^{-|x|}dx \) exists, define \( Q(C) = \int_C e^{-|x|}dx. \) Show that \( Q(C) \) is not a probability set function. What constant do we need to multiply the integrand by to make \( Q(C) \) a probability set function?

4. (1.3.10) Suppose we turn over cards simultaneously from two well shuffled decks of ordinary playing cards. We say we obtain an exact match on a particular turn if the same card appears from each deck; for example the queen of spades against the queen of spades. Let \( p_M \) equal the probability of at least one exact match.

(a.) Show that
\[
p_M = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots - \frac{1}{52!}.
\]

**Note:** There is a hint in the book.
(b.) Show that \( p_M \approx 1 - e^{-1} \).

(c.) * Suppose instead of requiring the exact same card for a match, we only require two cards to have the same rank (that is both kings). Let \( p'_M \) denote the probability of at least one match. What is \( p'_M \)?

5. Recall that the Borel \( \sigma \)-field \( \mathcal{B} \) is defined as follows: Let \( \mathcal{C} = \{(a,b) : a, b \in \mathbb{R} \cup \{\infty\}\} \). Then \( \mathcal{B} = \sigma(\mathcal{B}) \), the smallest \( \sigma \)-field containing \( \mathcal{C} \).

Define \( \mathcal{C}' = \{[a,b] : a, b \in \mathbb{R} \cup \{\infty\}\} \). Let \( \mathcal{B}' = \sigma(\mathcal{C}') \). Show that \( \mathcal{B} = \mathcal{B}' \).

**Hint:** It suffices to show that \( \mathcal{C} \subseteq \mathcal{B}' \) and \( \mathcal{C}' \subseteq \mathcal{B} \). Why?

6. (1.3.16) In a lot of 50 light bulbs, there are 2 bad bulbs. An inspector examines 5 bulbs, which are selected at random and without replacement.

(a.) Find the probability that at least one defective bulb being among the five.

(b.) Instead of five bulbs, the examiner chooses \( k \) bulbs at random and without replacement. How large must \( k \) be to ensure that the probability of finding at least one bad bulb exceeds \( \frac{1}{2} \).

(c.) Instead of choosing \( k \) bulbs at random and without replacement, the inspector chooses \( k \) bulbs at random, with replacement. How large must \( k \) be to ensure that the probability of finding at least one bad bulb exceeds \( \frac{1}{2} \).

**Suggested problems:** 1.2.3, 1.2.5, 1.2.6, 1.3.1, 1.3.9, 1.3.11-1.3.15 (not to be turned in).