Math 361, Problem set 11

Due 11/6/10

1. (3.4.32) Evaluate $\int_2^3 \exp(-2(x-3)^2)dx$ - without a calculator. Use the appendix table.

2. (3.4.19) Let the random variable X have a distribution that is $N(\mu, \sigma^2)$.

   (a) Does the random variable $Y = X^2$ also have a normal distribution?

   (b) Would the random variable $Y = aX + b$ have a normal distribution?

3. (3.4.22) Let $f(x)$ and $F(x)$ be the pdf and the cdf of a distribution of the continuous type such that $f'(x)$ exists for all x. Let the mean of the truncated distribution that has pdf $g(y) = f(y)/F(b), -\infty < y < b$, zero elsewhere, be equal to $-f(b)/F(b)$ for all real b. Prove that $f(x)$ is a pdf of a standard normal distribution.

4. (4.2.4) Let $X_1, \ldots, X_n$ be iid random varaibles with common pdf with $f(x) = e^{-x-\theta}$ for $x > \theta$, 0 elsewhere, where $-\infty < \theta < \infty$ is fixed. This pdf is called the shifted exponential. Let $Y_n = \min\{X_1, \ldots, X_n\}$. Prove that $Y_n \to \theta$ in probability, by obtaining the cdf and pdf of Y_n. (Note: if you can do it without obtaining the pdf and cdf of Y_n, that's fine too.)

5. (4.3.11) Let the random variable Z_n have a Poisson distribution with parameter $\mu = n$. Show that the limiting distribution of the random variable $Y_n = (Z_n - n)/\sqrt{n}$ is normal with mean zero and variance 1.

6. (4.3.7) Let X_n have a gamma distribution with parameter $\alpha = n$ and β where β is not a function of n. Let $Y_n = X_n/n$. Find the limiting distribution of Y_n. Hint: Find the mgf of Y_n (not bad as we found the mgf of X_n in class). Take the limit and figure out what kind of distribution leads to this new mgf. Hint 2: this is the MGF of a *constant* function. What constant? Why does this make sense?

7. (4.4.6) Let Y be $Bin(400, \frac{1}{5})$. Compute an approximate value of $\mathbb{P}(0.25 < Y/400)$.

8. (4.4.9) Let $f(x) = 1/x^2$ for $1 < x < \infty$, zero elsewhere, be the pdf of a random variable X. Consider a random sample of size 72 from the

distribution (i.e. 72 i.i.d. random variables $X_1, \ldots, X_{72}$). Compute approximately the probability that more than 50 of the observations of the random sample are less than 3.