Math 361, Problem Set 2

September 3, 2010

Due: 9/13/10

1. (1.3.11) A bowl contains 16 chips, of which 6 are red, 7 are white and 3 are blue. If four chips are taken at random and without replacement, find the probability that
   (a) each of the 4 chips is red
   (b) none of the four chips is red
   (c) there is at least one chip of each color.

2. (1.3.24) Consider three events C_1, C_2, C_3.
   (a) Suppose C_1, C_2, C_3 are mutually exclusive events. If $\mathbb{P}(C_i) = p_i$, for $i = 1, 2, 3$ what is the restriction on the sum p_1, p_2, p_3.
   (b) In the notation of Part (a), if $p_1 = 4/10$, $p_2 = 3/10$, and $p_3 = 5/10$ are C_1, C_2 and C_3 mutually exclusive?
   (c) Does it follow from your conclusion in part (b) that $\mathbb{P}(C_1 \cap C_2 \cap C_3) > 0$? Why or why not?

3. (1.4.7) A pair of 6-sided dice is cast until either the sum of seven or eight appears.
   (a) Show that the probability of a seven before an eight is 6/11.
   (b) Next, this pair of dice is cast until a seven appears twice (as a sum) or until each of a six and an eight have appeared at least once. Show that the probability of the six and eight occuring before two sevens is 0.546.

4. (1.4.4) A hand of 13 cards is to be dealt at random and without replacement from an ordinary deck of playing cards. Find the conditional probability that there are at least three kings in the hand given that the hand contains two kings.

5. (1.4.9) Bowl I contains 6 red chips and 4 blue chips. Five of these 10 chips are selected at random and without replacement and put into bowl II, which was initially empty. One chip is then drawn at random from bowl II. Given that this chip is blue, find the conditional probability that 2 red chips and 3 blue chips are transferred from bowl I to bowl II.

6. (The two drunks problem): Two drunks, one stupid and one smart wander back to their apartment after a night of drinking to by stymied at their door. They both have n keys, but are unable to tell which one unlocks their apartment.

   (a) The first, stupid drunk picks a key from his keychain uniformly at random repeatedly *with replacement* (i.e. he lets the key fall back with the rest of his keys if it doesn't work). Let A_k be the event that he successfully opens the door on the kth try. Compute $\mathbb{P}(A_k)$ for all k.

   (b) The smart drunk drops keys that don't work on the floor so he doesn't repeat bad keys. That is, he picks keys repeatedly *without replacement.* Let B_k be the event he successfully opens the door on the kth try. Compute $\mathbb{P}(B_k)$ for $k = 1, \ldots, n$. (Note: $\mathbb{P}(B_k) = 0$ for $k > n$. Why?)