Math 361, Problem set 3

Due 9/20/10

1. (1.4.21) Suppose a fair 6-sided die is rolled 6 independent times. A match occurs if side i is observed during the ith trial, $i = 1, \ldots, 6$.
   (a) What is the probability of at least one match during on the 6 rolls.
   (b) Extend part (a) to a fair n-sided die with n independent rolls. Then determine the limit of the probability as $n \to \infty$.

2. (1.4.32) Hunters A and B shoot at a target; their probabilities of hitting the target are p_1 and p_2 respectively. Assuming independent, can p_1 and p_2 be chosen so that

$$\mathbb{P}(0 \text{ hits}) = \mathbb{P}(1 \text{ hit}) = \mathbb{P}(2 \text{ hits})?$$

3. (1.5.1) Let a card be selected from an ordinary deck of playing cards. The outcome c is one of these 52 cards. Let $X(c) = 4$ if c is an ace, let $X(c) = 3$ if c is a king, $X(c) = 2$ if c is a king and $X(c) = 1$ if c is a jack. Otherwise $X(c) = 0$. Suppose $\mathbb{P}$ assigns a probability of $\frac{1}{52}$ to each outcome c. Describe the induced probability $\mathbb{P}_X(D)$ on the space $\mathcal{D} = \{0, 1, 2, 3, 4\}$ of the random variable X.

4. (1.5.9) Consider an urn which contains slips of aper each with one of the numbers $1, 2, \ldots, 100$ on it. Suppose there are i slips with the number i on it for $i = 1, 2, \ldots, 100$. E.g. there are 25 slips of paper with the number 25. Suppose one slip is drawn at random, let X be the number of the slip.
   (a) Show that X has pmf $p(x) = x/5050$, $x = 1, 2, 3, \ldots, 100$, zero elsewhere.
   (b) Compute $\mathbb{P}(X \leq 50)$.
   (c) Show that the cdf of X is $F(x) = [x]([x] + 1)/10100$ for $1 \leq x \leq 100$ where $[x]$ is the greatest integer in x (ie, $[100.12] = 100$.)

5. (1.5.10) Lte X be a random variable with space $\mathcal{D}$. For a sequence of sets $\{D_n\}$ in $\mathcal{D}$ show that

$$\{c : X(c) \in \bigcup_u D_n\} = \bigcup_n \{c : X(c) \in D_n\}$$

Use this to show that the induced probability $\mathbb{P}_X$ (see eq. 1.5.1) satisfies the third (additive) axiom of probability.

6. (1.6.2) Let a bowl contain 10 chips of the same shape and size. One, and only one, of these chips is red. Continue to draw chips from the bowl, one at a time and at random without replacement, until the red chip is drawn.

   (a) Find the pmf of X, the number of trials needed to draw the red chip

   (b) Compute $\mathbb{P}(X \leq 4)$.