Math 361, Problem set 4

Due 9/20/10

1. (1.4.26) Person A tosses a coin and then person B rolls a die. This is repeated independently until a head or one of the numbers $1, 2, 3, 4$ appears, at which time the game is stopped. Person A wins with the head, and B wins with one of the numbers $1, 2, 3, 4$. Compute the probability A wins the game.

2. (1.5.6) Let the probability set function $P_X(D)$ of the random variable X be $\mathbb{P}_X(D) = \int_D f(x)dx$, where $f(x) = \frac{2x}{9}$, for $x \in \mathcal{D} = \{x : 0 < x < 3\}$. Let $D_1 = \{x : 0 < x < 1\}$, $D_2 = \{x : 2 < x < 3\}$. Compute $\mathbb{P}_X(D_1) = \mathbb{P}(X \in D_1)$, $\mathbb{P}_X(D_2) = \mathbb{P}(X \in D_2)$ and $\mathbb{P}_X(D_1 \cup D_2) = \mathbb{P}(X \in D_1 \cup D_2)$.

3. (1.5.5) Let us select five cards at random and without replacement from an ordinary deck of playing cards.

   (a) Find the pmf of X, the number of hears in the hand.,

   (b) Determine $\mathbb{P}(X \leq 1)$.

4. A weighted coin, with head probability $\frac{1}{10}$ is flipped n times, where n is divisible by 10. Let X denote the number of heads flipped. Then the pmf of X is $p(k) = \binom{n}{k}(1/10)^k(9/10)^{n-k}$. Show which value of k this maximizes this. *Hint:* Look at the ratio: $p(k)/p(k+1)$. When k is small, this is less than one, when k is large, this is bigger than one. Find the value of k when $p(k)/p(k+1) \approx 1$. Why does this work?

5. (1.6.3) Cast a die a number of independent times until a six appears on the up side of the die.

   (a) Find the pmf $p(x)$ of X, the number of casts needed to obtain that first six.

   (b) Show that $\sum_{x=1}^{\infty} p(x) = 1$.

   (c) Determine $\mathbb{P}(X = 1, 3, 5, 7, \dots)$.

   (d) Find the cdf $F_X(x) = \mathbb{P}(X \leq x)$.