Math 361, Problem Set 2

October 26, 2010

Due: 11/1/10

1. (2.1.5) Given that the nonnegqative function $g(x)$ has the property that $\int_0^\infty g(x)dx = 1$, show that

$$f(x_1, x_2) = \frac{2g(\sqrt{x_1^2 + x_2^2})}{\pi\sqrt{x_1^2 + x_2^2}}, \quad 0 < x_1 < \infty \quad 0 < x_2 < \infty,$$

   zero elsewhere, satisfies the conditions for a pdf of two continuous-type random variables X_1 and X_2. *Hint: Use polar coordinates*

2. (2.1.8) Let 13 cards be taken, at random and without replacement, from an ordinary deck of playing cards. If X is the number of spades in these 13 cards, find the pmf of X. If, in addition Y is the number of heardts in these 13 cards, find the probability $\mathbb{P}(X = 2, Y = 5)$. What is the joint pmf of X and Y.

3. (2.1.14) Let X_1, X_2 be two random variables with joint pmf $p(x_1, x_2) = (1/2)^{x_1+x_2}$ for $x_i \in \{1, 2, 3, 4, \dots\}$ with $i = 1, 2$ and zero elsewhere. Determine the joint mgf of X_1, X_2. Show that $M(t_1, t_2) = M(t_1, 0)M(0, t_2)$.

4. (2.1.16) Let X and Y have the joint pdf $f(x, y) = 6(1 - x - y)$ for $x + y < 1, 0 < x, 0 < y$ and zero elsewhere. Compute $\mathbb{P}(2X + 3Y < 1)$ and $\mathbb{E}[XY + 2X^2]$.

5. (2.2.2) Let X_1 and X_2 have the joint pmf $p(x_1, x_2) = \frac{x_1 x_2}{36}$ for $x_1 = 1, 2, 3$ and $x_2 = 1, 2, 3$; zero elsewhere. Find first the joint pmf of $Y_1 = X_1 X_2$ and $Y_2 = X_2$, and then find the marginal pmf of Y_1.

6. (2.2.7) Use the formula (2.2.1) to find the pdf of $Y_1 = X_1 + X_2$, where X_1 and X_2 have the joint pdf $f_{X_1,X_2}(x_1, x_2) = 2e^{-(x_1+x_2)}$, $0 < x_1 < x_2 < \infty$, zero elsewhere.