

## Math 361, Problem set 9

Due 11/1/10

1. (2.2.3) Let  $X_1$  and  $X_2$  have the joint pdf  $h(x_1, x_2) = 2e^{-x_1-x_2}$ ,  $0 < x_1 < x_2 < \infty$ , zero elsewhere. Find the joint pdf of  $Y_1 = 2X_1$  and  $Y_2 = X_2 - X_1$ .
2. (2.3.2) Let  $f_{1|2}(x_1|x_2) = c_1x_1/x_2^2$ ,  $0 < x_1 < x_2$ ,  $0 < x_2 < 1$  zero elsewhere, and  $f_2(x_2) = c_2x_2^4$ ,  $0 < x_2 < 1$ , zero elsewhere, denote, respectively, the conditional pdf of  $X_1$  given  $X_2 = x_2$  and the marginal pdf of  $X_2$ . Determine
  - (a) The constants  $c_1$  and  $c_2$ .
  - (b) The joint pdf of  $X_1$  and  $X_2$ .
  - (c)  $\mathbb{P}(\frac{1}{4} < X_1 < 1/2 | X_2 = \frac{5}{8})$
  - (d)  $\mathbb{P}(1/4 < X_1 < 1/2)$
3. (2.3.5) Let  $X_1$  and  $X_2$  be two random variables such that the conditional distributions and means exist. Show that
  - (a)  $\mathbb{E}[X_1 + X_2 | X_2] = \mathbb{E}[X_1 | X_2] + X_2$
  - (b)  $\mathbb{E}[u(X_2) | X_2] = u(X_2)$ .
4. (2.3.9) Five cards are drawn at random and without replacement from an ordinary deck of cards. Let  $X_1$  and  $X_2$  denote, respectively, the number of spades and the number of hearts that appear in the five cards.
  - (a) Determine the joint pmf of  $X_1$  and  $X_2$
  - (b) Find the two marginal pmfs
  - (c) What is the conditional pmf of  $X_2$  given  $X_1 = x_1$ .

*Note:* First two parts are similar to what was on your last homework!

5. (2.3.11) Let us choose at random a point from the interval  $(0, 1)$  and let the random variable  $X_1$  be equal to the number which corresponds to that point. Then choose a point at random from the interval  $(0, x_1)$ , where  $x_1$  is the experimental value of  $X_1$ ; and let the random variable  $X_2$  be equal to the number which corresponds to this point.

- (a) Make assumptions about the marginal pdf  $f_1(x_1)$  and the conditional pdf  $f_{2|1}(x_2|x_1)$ .
- (b) Compute  $\mathbb{P}(X_1 + X_2 \geq 1)$ .
- (c) Find the conditional mean  $\mathbb{E}[X_1|x_2]$ .