

# Midterm Exam I

Math 361  
9/27/10

Name: \_\_\_\_\_

**Read all of the following information before starting the exam:**

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single  $8\frac{1}{2} \times 11$  sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Good luck!

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**1.** (*20 points*) A coin is flipped until there are two heads in a row, or three tails. Find the probability that the three tails comes first.

**2.** (*20 points*) Three integers are chosen with replacement from the first twenty integers. Find the probability that

**a.** (*10 pts*) Their sum is even

**b.** (*10 pts*) Their product is even.

**3.** (20 points)  $X$  is a continuous random variable with cdf  $F_X(x) = 1 - (1 - x)^2$  for  $0 \leq x \leq 1$ , with  $F_X(x) = 0$  for  $x \leq 0$ , and  $F_X(x) = 1$  for  $x \geq 1$ .      **a.** (10 pts) Find the pdf of  $X$ .

**b.** (10 pts) Let  $Y = 2(X + 1)^2$ . Find the cdf of  $Y$ .

**4.** (*20 points*) A random integer  $N$  from 1 to 10 is chosen uniformly at random.

**a.** (*10 pts*) The random variable  $X$  denotes the number of distinct prime factors of  $N$ . (So if  $N = 8 = 2^3$ ,  $X = 1$ ). Find the pmf of  $X$ .

**b.** (*10 pts*) Another random integer  $N'$  from 1 to 10 is also chosen uniformly at random. Find the probability that  $N > 2N'$ .

**5.** (*20 points*) Is it possible for two events  $A$  and  $B$  with  $\mathbb{P}(A), \mathbb{P}(B) > 0$  to be both mutually exclusive and independent. Why or why not?

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(please do not remove this page from the test packet)