**Midterm Exam I**

Read all of the following information before starting the exam:

- **READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!**
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- **READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!**
- Good luck!

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1. (20 points) A coin is flipped until there are two heads in a row, or three tails. Find the probability that the three tails comes first.
2. (20 points) Three integers are chosen with replacement from the first twenty integers. Find the probability that

a. (10 pts) Their sum is even

b. (10 pts) Their product is even.
3. (20 points) $X$ is a continuous random variable with cdf $F_X(x) = 1 - (1 - x)^2$ for $0 \leq x \leq 1$, with $F_X(x) = 0$ for $x \leq 0$, and $F_X(x) = 1$ for $x \geq 1$.

a. (10 pts) Find the pdf of $X$.

b. (10 pts) Let $Y = 2(X + 1)^2$. Find the cdf of $Y$. 
4. (20 points) A random integer $N$ from 1 to 10 is chosen uniformly at random.
   
   a. (10 pts) The random variable $X$ denotes the number of distinct prime factors of $N$. (So if $N = 8 = 2^3$, $X = 1$). Find the pmf of $X$.

   b. (10 pts) Another random integer $N'$ from 1 to 10 is also chosen uniformly at random. Find the probability that $N > 2N'$. 
5. (20 points) Is it possible for two events \(A\) and \(B\) with \(P(A), P(B) > 0\) to be both mutually exclusive and independent. Why or why not?