

Midterm Exam II

Math 361
9/27/10

Name: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single $8\frac{1}{2} \times 11$ sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Good luck!

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1. (20 points) Suppose random variables X_1, X_2 and X_3 have joint pdf $f(x_1, x_2, x_3) = \frac{x_1+x_2+x_3}{3}$ for $0 < x_1 < 1, 0 < x_2 < 1, 0 < x_3 < 1$.

a. (10 pts) Suppose $Y_1 = X_1, Y_2 = X_1 + X_2, Y_3 = X_1 + X_2 + X_3$. Compute the joint pdf of Y_1, Y_2, Y_3 .

b. (10 pts) Compute $\mathbb{E}[Y_2]$.

2. (*20 points*) Prescott Pharmaceuticals has developed a new test for the disease Horrible type A. Only $\frac{1}{1000}$ people is infected with Horrible type A, and the test is correctly positive on sick people 98% of the time. Unfortunately 2% of the time, it returns positive on a well person. Let S denote the event a person is sick, and P denote the event a test is positive.

a. (*10 pts*) Determine $\mathbb{P}(S|P)$.

b. (*10 pts*) Suppose instead, the false positive rate was p instead of 2% (with all other parameters equal). How small must p be so that $\mathbb{P}(S|P) = .99$?

3. (*20 points*) A uniformly random variable X is chosen in $(0, 1)$. Then $X_1, X_2, X_3, \dots$ are independent random variables chosen uniformly in $(0, X)$. Let $Y_n = \max\{X_1, \dots, X_n\}$. Show that $Y_n \rightarrow_p X$.

4. (*20 points*)

a. (*10 pts*) Jim didn't study for his multiple choice probability test and is uniformly and independently randomly choosing one of the four answers for each of the 200 problems. Let X denote his percentage (/100) on the exam. Estimate

$$\mathbb{P}(X > 30).$$

b. (*10 pts*) A lightbulbs life in years is given by the exponential distribution $f(x) = \frac{1}{2}e^{-x/2}$. Mary buys a pack of 64 lightbulbs at Costco, and uses them one by one. Let X_i denote the life of the i th bulb, then their total life is $Y = \sum_{i=1}^{64} X_i$. Estimate

$$\mathbb{P}(Y < 110)$$

- 5.** (*20 points*) A hand of 6 cards are drawn from a deck without replacement.
- a.** (*10 pts*) Find the pmf for the number of clubs in the hand.

b. (*10 pts*) Find the probability that the hand contains a four of a kind (four cards of the same rank - no restrictions on the other two cards).

6. (*20 points*)

a. (*10 pts*) Show that $\text{Var}(\alpha X) = \alpha^2 \text{Var}(X)$.

b. (*10 pts*) Show that $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ if X and Y are independent.

7. (*20 points*) A dice is rolled until a 6 is rolled or two fives (not necessarily in a row) are rolled. Find the probability that the two fives come first.

8. (*20 points*) A dice is rolled, let X be the result of that roll. Then X cards are drawn from a deck, let Y denote the number of hearts in that deck.

a. (*10 pts*) Determine

$$\mathbb{P}_{Y|X}(y|x)$$

(Hint: No need to find $p_{X,Y}(x,y)$ to do this part.)

b. (*10 pts*) Determine

$$\mathbb{P}_{X,Y}(x,y).$$

(Hint: Use part (a).)