Midterm Exam II

Math 361
9/27/10

Name: ________________________________

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Good luck!

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1. (*20 points*) Thirteen cards are drawn at random without replacement from a standard deck. Let X denote the number of face cards drawn (kings, queens or jacks) and let Y denote the number of spades drawn.

a. (*10 pts*) Find the marginal pmfs of X and Y.

b. (*10 pts*) Find the joint pmf $p_{X,Y}(x, y)$.

2. (*20 points*) **a.** (*10 pts*) Suppose X_n has the Gamma distribution $\Gamma(n, \beta)$. Explain why

$$\frac{X_n - \frac{n}{\beta}}{\frac{\sqrt{n}}{\beta^2}} \to_p N(0,1).$$

b. (*10 pts*) Prove the weak law of large numbers: If X_n are iid with $\mathbb{E}[\mathrm{X}] = \mu$ and $\mathrm{Var}(X) = \sigma^2 < \infty$, let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then

$$\bar{X}_n \to_p \mu$$

3. (*20 points*) **a.** (*10 pts*) Let X, Y and Z be random variables with joint pdf $f(x, y, z) = e^{-x-y-z}$ for $x > 0, y > 0, z > 0$. Compute

$$\mathbb{P}(\mathrm{X} < \mathrm{Y} < \mathrm{Z} | \mathrm{Z} > 1).$$

b. (*10 pts*) Are X, Y, Z independent? Explain.

4. (*20 points*) **a.** (*10 pts*) Is it possible for two events A and B to be independent and mutually exclusive? Why or why not?

b. (*10 pts*) Independent events A, B and C have $\mathbb{P}(\mathrm{A}) = .9$, $\mathbb{P}(\mathrm{B}) = .3$ and $\mathbb{P}(\mathrm{C}) = .2$. Find

$$\mathbb{P}((\mathrm{A} \cup \mathrm{B}) \cap \mathrm{C}^{\mathrm{c}})$$

5. (*20 points*) A random angle X is chosen according to pdf $f_X(x) = \frac{2}{\pi^2}x$ for $0 < x < \pi$, 0 otherwise. Let $Y = \cos(X)$.

a. (*10 pts*) Compute $\mathbb{E}$[Y].

b. (*10 pts*) Find the pdf $f_Y(y)$.

Note: $\frac{d}{dx}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}$.

6. (*20 points*) **a.** (*10 pts*) Suppose X is a $N(2,3)$ random variable. Determine

$$\mathbb{P}(\mathrm{X} > 1).$$

b. (*10 pts*) Y is a $Bin(320, \frac{1}{8})$ random variable. Estimate $\mathbb{P}(\mathrm{X} < 35)$.

7. (*20 points*) There are three widget factories construction widgets. The first constructs 1000 in a day, with 5% defective. The second constructs 10000 in a day, but only 1% are defective. The third constructs 4000 a day, but 10% are defective.

a. (*10 pts*) One of the 15000 widgets constructed today is chosen uniformly at random. It is defective. What is the probability it came from the second factory?

b. (*10 pts*) How many widgets must be selected at random (with replacement) from the 11000 constructed today so that the probability of finding a defective one is at least 0.5?

8. (*20 points*)
Random variables X and Y have conditional pdf $f_{X|Y}(x|y) = e^{-x+2y}$ for $x > 2y$ and Y has pdf $f_Y(y) = 2e^{-2y}$ for $y \geq 0$.

a. (*10 pts*) Find $\mathbb{E}[\mathrm{X}|\mathrm{y}]$.

b. (*10 pts*) Compute the pdf of $\mathbb{E}[\mathrm{X}|\mathrm{Y}]$.

9. (*20 points*) Three integers are chosen with replacement from the first twenty integers. Find the probability that **a.** (*8 pts*) Their sum is even.

b. (*7 pts*) Their product is even

c. (*5 pts*) Their product is at least 10.

10. (*20 points*) Recall the MGF for a Poisson random variable X with parameter λ is $M_X(t) = e^{\lambda(e^t-1)}$. **a.** (*10 pts*) Use this to find $\mathrm{Var}(X)$

b. (*10 pts*) Find $\mathbb{E}[\mathrm{X}^4]$.