Math 362, Problem set 2

Due 2/9/10

1. (5.2.17) Let $Y_1 < Y - 2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with pdf $f(x) = 2x$, $0 < x < 1$, zero elsewhere.

   (a) Find the joint pdf of Y_3 and Y_4.

   (b) Find the conditional pdf of Y_3, given $Y_4 = y_4$.

   (c) Evaluate $\mathbb{E}[Y_3|y_4]$.

2. (5.4.9) Let $\bar{X}$ denote hte mean of a random sample of size 25 from a gamma-type distribution with $\alpha = 4$ and $\beta > 0$. Use the Central Limit theorem to find an approximate 0.954 confidence interval for μ, the mean of the gamma distribution.

   Hint: Use the random variable $(\bar{X} - 4\beta)/\sqrt{4\beta^2/25}$

3. (5.4.13) Let $Y_1 < Y_2 < \cdots < Y_n$ denote hte order statistics of a random sample of size n from a distribution that has pdf $f(x) = 3x^2/\theta^3$, $0 < x < \theta$, zero elsewhere.

   (a) Show that $\mathbb{P}(c < Y_n/\theta < 1) = 1 - c^{3n}$ where $0 < c < 1$.

   (b) If n is 4 and if the observed value of Y_4 is 2.3, what is a 95% confidence interval for θ.

4. (5.4.16) When 100 tacks were thrown on a table, 60 of them landed point up. Obtain a 95% confidence interval for the probability that a tack of this type will land point up. Assume independence.

5. (5.4.14) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where both parameters μ and σ^2 are unknown. A confidence interval for σ^2 can be found as follows: We know $(n-1)S^2/\sigma^2$ is a random variable with a $\chi^2(n-1)$ distribution. Thus we can find constants a and b so that $\mathbb{P}((n-1)S^2/\sigma^2 < b) = 0.975$ and $\mathbb{P}(a < (n-1)S^2/\sigma^2 < b) = 0.95$.

   (a) Show that this second probability statement can be written as

   $$\mathbb{P}((n-1)S^2/b < \sigma^2 < (n-1)S^2/a) = 0.95$$

(b) If $n = 9$ and $s^2 = 7.93$ (here s^2 is the actual value of S^2 given data), find a 95% confidence interval for σ^2.

(c) If μ is known, how would you modify the preceding procedure for finding a confidence interval for σ^2.

6. (3.6.2) Let T have a t-distribution with 14 degrees of freedom. Determine b so that $\mathbb{P}(-b < T < b) = 0.9$. Use Table IV.