Math 362, Problem set 4

Due 2/23/10

Note: If you turn HW in on 2/21, I will return it for you by 2/23 for the purpose of studying for the exam.

1. (5.5.13) Let p denote the probability that, for a particular tennis player, the first serve is good. Since $p = .4$, this player decided to take lessons in order to increase p. When the lessons are completed, the hypothesis $H_0 = p = .4$ will be tested against $H_1 : p > .4$ based on $n = 25$ trials. Let y equal the number of first serves that are good, and let the critical region be defined by $C = \{y : y \geq 13\}$

   (a) Determine $\alpha = \mathbb{P}_{p=.4}(Y \geq 13)$.

   (b) Find $\beta = \mathbb{P}_{p=.6}(Y < 13)$. That is $1 - \beta$ is the power at $p = 0.6$,.

2. (5.6.2) Consider the power function $\gamma(\mu)$ and it's derivative given by (5.6.5). Show that $\gamma'(\mu)$ is strictly negative for $\mu < \mu_0$, and strictly positive for $\mu > \mu_0$. What does this mean about $\gamma(\mu)$?

3. (5.6.9) In exercise 5.4.14, (on HW 2) we found a confidence interval for the variance σ^2 using the variance S^2 of a random sample of size n arising from $N(\mu, \sigma^2)$, where the mean μ is unknown. In testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$, use the critical region defined by $(n-1)S^2/\sigma_0^2 \geq c$. That is, reject H_0 and accept H_1 if $S^2 \geq c\sigma_0^2/(n-1)$. If $n = 13$ and the significance level $\alpha = 0.025$, determine c.

4. (5.7.3) A die was cast $n = 120$ time independent times, and the following data resulted:

Spots Up	1	2	3	4	5	6
Frequency	b	20	20	20	20	$40 - b$

   If we use a chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at the 0.025 significance level.

5. (5.7.7) A certain genetic model suggest that the probabilities of a particular trinomial distribution are, respectively, $p_1 = p^2$, $p_2 = 2p(1-p)$ and $p_3 = (1-p)^2$, where $0 < p < 1$. If X_1, X_2, X_3 represent the respective frequencies in n independent trials, explain how we could check on the adequacy of the genetic model.

6. (5.8.3). Suppose X is a random variable with the pdf $f_X(x) = b^{-1} f((x-a)/b)$, where $b > 0$. Suppose we can generate observations from $f(z)$. Explain how we can generate observations from $f_X(x)$.

Remark: I promised some test-like questions, but the book questions are pretty good in this regard this time - especially 1, 3, 4. Even problems like 5 written a bit less open endedly is okay. 6, I would try and make more concrete. As I hit sections where the book problems are less suitable for exam problems, though, I will try and write some of my own problems. Please give me feedback as to whether how the homework problems are going for you as we head towards this exam.