Group Theory and Its Neighboring Disciplines

Conference in honor of Jon Hall's 75th birthday

August 16-18, 2024
University of Denver

Contents

Overview

The conference will honor and highlight the work of Professor Jon Hall on the occasion of his 75th birthday.

Topics covered by the conference include group theory, representation theory, Lie theory, group cohomology, non-associative algebraic structures, computational group theory, combinatorics, discrete geometry, and coding theory.

Speakers and Invited Participants:

Monther Alfuraidan (King Fahd University of Petroleum & Minerals)
Barbara Baumeister (University of Bielefeld)
Hans Cuypers (Technische Universiteit Eindhoven)
Ales Drapal (Charles University)
Clara Franchi (Catholic University of Sacred Heart)
Kenneth Johnson (Penn State Abington)
Bill Kantor (University of Oregon)
Michael Kinyon (University of Denver)
Mark Lewis (Kent State University)
Richard Lyons (Rutgers University)
Mario Mainardis (University of Udine)
William Martin (Worcester Polytechnic Institute)
Dan Nakano (University of Georgia)
Guillermo Nunez Ponasso (Worcester Polytechnic Institute)
Cheryl Praeger (University of Western Australia)
Gerardo Raggi (UNAM Morelia)
Julianne Rainbolt (Saint Louis University)
Igor Rapinchuk (Michigan State University)
Bernardo Rodrigues (University of Pretoria)
Yoav Segev (Ben-Gurion University)
Sergey Shpectorov (University of Birmingham)
Jonathan Smith (Iowa State University)
Steve Smith (University of Illinois at Chicago)
Justin Stevenson (Iowa State University)
Simon Thomas (Rutgers University)
Pham Tiep (Rutgers University)
Richard Weiss (Tufts University)
Tingyao Xiong (Radford University)

Sponsors:

The conference is supported by a grant from the National Security Agency and by the Department of Mathematics at the University of Denver.

Program

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	Some groups of order n^3
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4:05 - 4:30	Igor Rapinchuk
	Reductive groups over polynomial rings and buildings
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	Weakly primitive axial algebras

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Simple group orders and prime-power-covering subgroups

09:35 - 10:00 Hans Cuypers

Graphs, axial algebra and their automorphism groups

Coffee break

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Restricting Rational Modules to Frobenius Kernels

11:05 - 11:30 Tingyao Xiong

Generalizations of wreath product identities via Garsia-Gessel bijections

11:40 - 12:05 Barbara Baumeister

Covering a group by the conjugates of a coset

Lunch break

2:00 - 2:25 Mark L. Lewis

Semi-extraspecial groups with automorphisms of large order

2:35 - 3:00 Gerardo Raggi

Biset functors and global representation rings

Coffee break

3:30 - 3:55 Justin Stevenson

Character quasigroups: A generalization of Pontryagin duality

4:05 - 4:30 Sergey Shpectorov

On the structure of algebras of Jordan type

4:40 - 5:05 Clara Franchi

Axial algebras of Monster type

Sunday, August 18, 2024

09:00 - 09:25 William J. Martin

Galois fusions of association schemes

09:35 - 10:00 Pham Tiep (pre-recorded)

Asymptotic results on Thompson's conjecture

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10:30 - 10:55 Ales Drapal

Solvability and nilpotency of Moufang loops

11:05 - 11:30 Tathagata Basak

Hyperbolic lattices from strongly regular graphs

Coffee break

12:00 - 12:25 Kenneth Johnson

Questions arising in nonassociative algebra

12:35 - 1:05 Michael Kinyon

Loops with squares in two nuclei

Titles and Abstracts

Characterizing the 6-cube: a journey through Smiths theorem and distance-transitive graphs

Monther Alfuraidan

King Fahd University of Petroleum & Minerals

In this talk, we explore the seminal work of Derek Smith and its profound implications on the classification of distance-transitive graphs, focusing specifically on the unique structure of the 6-cube. By revisiting Smith's Theorem, we delineate a pathway through which most distance-regular graphs are reduced to their primitive cases, highlighting the critical role of imprimitivity in understanding complex graph structures. Our analysis confirms the 6-cube as the sole example in one of the exceptional cases outlined by Smith's classification, showcasing its distinctive properties as a distance-transitive graph. This presentation, based on joint work with J. I. Hall, not only honors the mathematical legacy of Prof. Jon Hall but also underscores the vibrant interplay between group theory and graph theory, offering insights into the symmetry and regularity that govern these mathematical landscapes.

Hyperbolic lattices from strongly regular graphs

Tathagata Basak

Iowa State University

Given a strongly regular graph, we'll define lattice of signature $(n, 1)$. The construction is analogous to definition of root lattices, given a simply laced dynkin diagram. In many examples these lattices have interesting hyperbolic reflection groups. For instance, the 275 vertex McLaughlin graph produces a signature (21, 1) lattice on which the McLaughlin group naturally acts and whose reflection group contains 275 reflections that braid or commute according to the McLaughlin graph. A variant of this construction works for certain regular bipartite graphs and a variant of the construction produces hyperbolic hermitian lattices over rings of integers of some quadratic number fields. In particular, we want to mention two examples over the ring of Eisenstein integers. One reflection group, in $U(4, 1)$, is related to the fundamental group of moduli space of cubic surfaces. Another, in $U(13, 1)$, is conjecturally related to the monster simple group.

Covering a group by the conjugates of a coset

Barbara Baumeister

Universität Bielefeld

Let G be a finite group. It is not possible to cover a group by the conjugates of a proper subgroup of G. If G acts doubly transitive on the coset space G/A, then for every proper coset Ax of A in G, the conjugates of Ax cover the group G. Does this characterize the doubly transitive permutation groups? I will report on recent advances. This is joint work with Gil Kaplan, Dan Levy and Hung Tong-Viet.

Graphs, axial algebra and their automorphism groups

Hans Cuypers

Mathematics at Technische Universiteit Eindhoven

Jonathan Hall is a master in using graph theoretical and geometric tools in order to solve problems in algebra. He together with Sergey Shpectorov and Felix Rehren introduced a class of algebras generated by special idempotents called axial algebras. These algebras are very interesting through their connection with various finite simple groups, including the Monster and 3-transposition groups. As shown by Shpectorov et al., the finite dimensional axial algebras do have a finite automorphism group, provided none of the generating idempotents has an eigenvalue 1/2. But which finite groups can occur as automorphism group of an axial algebra?

We introduce a class of axial algebras related to graphs. We determine their fusion laws, prove them to be simple in almost all cases, and determine their automorphism group under some conditions on the valencies and girth of the graph. A construction of a class of these graphs with prescribed automorphism group enables us to construct for each group G infinitely many simple (axial) algebras (with a fixed fusion law) such that the automorphism group of the algebra is isomorphic to G.

Solvability and nilpotency of Moufang loops

Ales Drapal
Charles University

Modern theory of Moufang loops starts with seminal works of Glauberman. Another mighty step forward was the connection to groups with triality as elaborated and advanced by Doro. A contemporary introduction to this approach may be found in two survey papers of Jonathan Hall and in his monograph..
The talk will focus on these recent results: (1) A direct proof that Moufang p-loops are nilpotent. (2) A proof that in finite Moufang loops the solvability implies congruence solvability. (3) A proof that the multiplication group of a finite solvable Moufang loop is solvable.
Some remarks: (1) was proved by Glaubermen first for odd p and then, in a separate paper, for $p = 2$. By a recent observation of Jonathan Hall (personal communication) it is possible to modify and extract parts of survey paper of his to get another short proof of the fact. The alternative proof I will mention proceeds in a completely different way and relies upon a result on relative multiplication groups that is also crucial for (2) and (3).
Congruence solvability is a notion of universal algebra that was transposed to loop theory by Stanovsky and Vojtechovsky. We now know that a normal abelian subgroup S of a Moufang loop Q yields an abelian congruence if and only if $(sx)t = s(xt)$ whenever x in Q and s, t in S. The induced refiment of normal series with abelian factors made possible to prove the result of (3) (a fact that was conjectured for decades but for which the machinery was missing).

Axial algebras of Monster type

Clara Franchi
Catholic Univerity of Sacred Heart

Extending earlier work by Ivanov on Majorana algebras, axial algebras of Monster type were introduced in 2015 by Hall, Rehren, and Shpectorov in order to axiomatise some key features of certain classes of algebras related to large families of finite simple groups, such as the weight-2 components of OZ-type vertex operator algebras, Jordan algebras, and Matsuo algebras. In this talk I'll present the current status of the classification of the 2-generated primitive axial algebras of Monster type.

Questions arising in nonassociative algebra

Kenneth Jonson
Penn State Abington

I will discuss various questions arising from my book: Group matrices and Group Determinants

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Some groups of order n^3

William Kantor
University of Oregon

I will discuss the relationship between some groups and a standard combinatorial conjecture.

Loops with squares in two nuclei

Michael Kinyon
University of Denver

The varieties of loops (quasigroups with identity elements) of Bol-Moufang type are defined by identities with the following properties: (i) they involve three distinct variables, each occurring on both sides of the equal sign; (ii) the variables occur in the same order on both sides; (iii) exactly one of the variables appears twice on both sides. For example, the identity $(xy)(zx) = x((yz)x)$ is of Bol-Moufang type and defines the variety of Moufang loops. (This is half of the rationale for the name "Bol-Moufang type").

Various interesting varieties of loops (interesting, at least, to quasigroup theorists) – such as Moufang loops, Bol loops, C loops and others – can be defined by identities of Bol-Moufang type. There are others which, up until now, have not been garnered much interest because there does not seem to be much that can be said about them. For instance, the variety of loops with *left nuclear squares* is defined by $(xx)(yz) = ((xx)y)z$; essentially nothing interesting can be proven about these or the similarly defined varieties of loops with middle nuclear or right nuclear squares.

It turns out however, that one can say interesting things about the pairwise intersection of these varieties (from whence comes the title of this talk). This is a bit surprising because on the face of it, the condition that squares associate with all other elements in certain positions does not seem like a very strong property. In particular, it turns out that in such loops, the intersection of the corresponding nuclei is a normal subloop. In addition, we consider the subvariety of loops with *central* squares. In such loops, having the automorphic inverse property (AIP) $(xy)\backslash 1 = (x\backslash 1)(y\backslash 1)$ is equivalent to endomorphic squaring $(xy)^2 = x^2y^2$. In the finite case, such loops decompose into a direct product into an abelian group of odd order and a loop consisting of elements of order a power of 2. This is analogous to what happens in other Bol-Moufang type varieties in the presence of the AIP.

Semi-extraspecial groups with automorphisms of large order

Mark L. Lewis

Kent State University

(This is joint work with Sofia Brenner and Rachel Camina.) Let G be a semi-extraspecial group so that $|G : Z(G)| = p^{2a}$ so that G has an automorphism of order $p^{2a} - 1$ that transitively permutes the cosets of $G/Z(G)$. We prove that G must be isomorphic to a Sylow p-subgroup of $\mathrm{SU}_3(p^{2a})$ and when p is odd, this implies that G is isomorphic to the more familiar Sylow p-subgroup of $\mathrm{SL}_3(p^a)$.

Majorana representations of finite groups

Mario Mainardis

University of Udine

Majorana Representations have been introduced by A. A. Ivanov in order to give an axiomatic framework for studying the embeddings of a group into the Monster group. I shall give an account of the current status of the research and of the methods involved.

Galois fusions of association schemes

William J. Martin

Worcester Polytechnic Institute

Motivated by the study of Delsarte designs in finite groups, Jesse Lansdown and I studied subspaces of Bose-Mesner algebras corresponding to subgroups of the Galois group of an association scheme. In this talk, I will focus on the conjugacy class scheme of a finite group; this is essentially the center of the group algebra, viewed inside the right regular representation. I will briefly discuss how this study of subalgebras informs our study of codes and designs in these association schemes.

Restricting rational modules to Frobenius kernels

Daniel Nakano

University of Georgia

Let G be a connected reductive group over an algebraically closed field of characteristic $p > 0$. Given an indecomposable G-module M, one can ask when does it remains indecomposable upon restriction to the Frobenius kernel G_r, and when its G_r-socle is simple (the latter being a strictly stronger condition than the former). In this talk, we investigate these questions for G having an irreducible root system of type A.

Using Schur functors and inverse Schur functors as our primary tools, we develop new methods of attacking these problems, and in the process obtain new results about classes of Weyl modules, induced modules, and tilting modules that remain indecomposable over G_r. A key part of obtaining these results is to use the representation theory of the symmetric group.

This talk represents joint work with Christopher Bendel, Cornelius Pillen and Paul Sobaje.

Simple group orders and prime-power-covering subgroups

Cheryl Praeger

University of Western Australia

In 1992 Laci Pyber showed that the order of a nonabelian simple group T is bounded above by $c^{f(m)}$ where$c$ is a positive constant, m is the number of $Aut(T)$-classes in T and $f(m) = (\log m)^2 \cdot \log\log m$. In 1994 I used this result to attack a conjecture about covering subgroups of a finite group G, that is a proper subgroup U of G containing a representative of each $Aut(G)$-class of elements of G. The conjecture is that $|G : U|$ is bounded above by a function of $|Out(G)|$, and has a number theoretic interpretation for Kronecker classes of algebraic number fields. Using Pybers result I was able to prove this in the case where U is maximal in G. Michael Giudici, Luke Morgan and I have just succeeded to prove a stronger form of this conjecture (related to relative Brauer groups of algebraic number fields) for maximal subgroups U of G: namely, if U contains a representative of each $Aut(G)$-class of elements of G of prime power order, then $|G : U|$ is bounded by a function of $|Out(G)|$. To do this we proved a 30 year old conjecture for nonabelian simple groups T (a strong version of Pybers result): there is a function f such that $|T| \leq f(m)$ where, for all primes p, the number of $Aut(T)$-classes of p-elements contained in T is at most m.

Biset functors and global representation rings

Gerardo Raggi

National Autonomous University of Mexico

In this talk we talk about biset functors for finite groups and give as some examples including the global representation ring.

Reductive groups over polynomial rings and buildings

Igor Rapinchuk

Michigan State University

Suppose G is a reductive algebraic group over a field k. We will show how the Fixed Point Theorem for actions of finite groups on affine buildings can be used to prove several results concerning the group scheme $Gx_k k[t]$ obtained by base change to the polynomial ring in one variable. In particular, we will present a short proof of the Raghunathan-Ramanathan Theorem about torsors over the affine line and discuss a statement concerning the finite subgroups of $G(k[t])$. This talk is based on results obtained jointly with P. Abramenko, V. Chernousov, and A. Rapinchuk.

2-Modular representations of the Conway simple groups as binary codes

Bernardo Rodrigues

University of Pretoria

In the talk we introduce and discuss an elementary tool from representation theory of finite groups for constructing linear codes invariant under a given permutation group G. The tool gives theoretical insight as well as a recipe for computations of generator matrices and weight distributions. In some interesting cases a classification of code vectors under the action of G can be obtained. As explicit examples we examine binary two-weight codes related to the 2-modular reduction of the Leech lattice and Conway groups

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Weakly primitive axial algebras

Yoav Segev

Ben-Gurion University

We consider a noncommutative, nonassociative algebras A over a field F of char. not 2. An **axis** is a left semisimple idempotent a which is also right semisimple, such that L_a (left multiplication) commutes with R_a. Thus an eigenvalue for a is a pair $(\mu, \nu) \in F \times F$. The axis a is **weakly primitive** if its $(1,1)$ eigenvectors are Fa.

We write $A_{\mu,\nu}(a)$ for the subspace of (μ,ν)-eigenvectors. a is an S-axis, where $S \subset F \times F$, if all (paired) eigenvalues of a belong to S. We specifically require that $(0,0), (1,1) \notin S$. Write $A_1(a)$ for $A_{1,1}(a)$, $A_0(a)$ for $A_{0,0}(a)$, and $A_{\mu,\nu}$ for $A_{\mu,\nu}(a)$.

An S-axis a **satsifies the fusion rules** if: (1) $A_{\mu,\nu}A_{\mu',\nu'} = 0$, for all distict $(\mu,\nu), (\mu',\nu') \in S$. (2) $A^2_{\mu,\nu} \subseteq A_1 + A_0$, for all $(\mu,\nu) \in S$. (3) $A_1 + A_0$ is a subalgebra of A. (4) $(A_1 + A_0)A_{\mu,\nu} \subseteq A_{\mu,\nu}$ and $A_{\mu,\nu}(A_1 + A_0) \subset A_{\mu,\nu}$.

We obtain plenty of information about A generated by axes a, b; and if both satisfy the fusion rules we are able to give full classification under certain additional conditions.

This is joint work with Louis Rowen.

On the structure of algebras of Jordan type

Sergey Shpectorov

University of Birmingham

The class of algebras of Jordan type η was introduces by Hall, Rehren and Shpectorov in 2015. Examples of such algebras include the Matsuo algebras, arising for all values of η and related to 3-transposition groups, and, of course, Jordan algebras for $\eta = \frac{1}{2}$.

In the talk, we will review the history of this interesting class of algebras, from the initial 2015 classification of the case $\eta \neq \frac{1}{2}$ to the recent results on the solid subalgebras and components in algebras of Jordan type half, which completely change the paradigm and lead to new examples.

Unary and binary triality

Jonathan D. H. Smith

Iowa State University

The phenomenon of duality is pervasive throughout mathematics, for example as the reversal of arrows in categories, or passing between geometry and algebra. As inversion, it is the symmetry of the language of groups.
The phenomenon of triality is much rarer. It comes in two closely related forms, which may be disambiguated as unary and binary triality. Unary triality has been a major theme of Jonathan Hall's work.
The current talk will focus on binary triality, as the symmetry of the language of equational quasigroups. We will see how binary triality appears in the theory of quantum quasigroups, which provide a self-dual unification of quasigroups and Hopf algebras. In particular, we may ask which Hopf algebras exhibit binary triality.

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50 years of mathematical friendship with Jon Hall

Stephen D. Smith

University of Illinois at Chicago

Jon and I were grad students together at Oxford in the early 70s. I'll look back at some of the high points of the time since then.

Character Quasigroups: A generalization of Pontryagin duality

Justin Stevenson

Iowa State University

Pontryagin duality provides a powerful tool for analyzing the structure and properties of locally compact abelian groups, in particular finite abelian groups. Now a similar thing can be done to non-abelian groups via the construction of character quasigroups. In this talk I give a brief overview of the necessary concepts and results from representation theory which underpins the existence of character quasigroups. I will then discuss the dihedral groups and the classification of their balanced character groups.

Invariant Random Subgroups and Characters of Locally Finite Groups

Simon Thomas

Rutgers University

In 2012, Vershik pointed out that the indecomposable characters of the group of finitary permutations of the natural numbers are closely connected with its ergodic invariant random subgroups; and he suggested that this should also be true of various other locally finite groups. In this talk, I will present a brief introduction to this topic and I will discuss some of the many open problems.

Asymptotic results on Thompson's conjecture

Pham Tiep

Rutgers University

John Thompson conjectured in 1982 that every finite non-abelian simple group G admits a conjugacy class C such that $G = C^2$. Previous results, most notably of Ellers–Gordeev (1998), show that the conjecture holds for all G except possibly for finite simple groups of Lie type over a field of less than 9 elements. We will report on recent joint work with Michael Larsen, proving the conjecture for several families of sufficiently large simple classical groups.

Tits polygons

Richard Weiss

Tufts University

The notion of a Tits polygon generalizes the notion of a Moufang polygon. There is a natural construction that produces Tits polygons from a given spherical building and a matching Tits index. These are the Tits polygons of index type. We will describe ongoing efforts to understand the structure of these Tits polygons. This is joint work with Bernhard Mühlherr.

Generalizations of wreath product identities via Garsia-Gessel bijections

Tingyao Xiong
Radford University

Garsia-Gessel bijections have been proven to be very powerful in obtaining generating functions of multivariate distributions. In 2011, Biaogioli and Zeng applied Garsia-Gessel bijection to successfully derive four and six variate distributions by defining ""Biaogiolo-Zeng"" partial ordering, or simply BZ ordering, on the set of wreath product $Z_r \wr S_n$. In this paper, we can prove that BZ's identities can be generalized to partial orders based on colors, for instance, the order $1^r <? < n^r < 1^(r-1) <? < n^(r-1) <? < 1^1 <? < n^1 < 0 < 1 < n$ defined by Adin-Roichman. We can prove that under the new partial order, the multivariate distribution identities can be significantly simplified, and can be more compatible with the classical four-variate distribution functions established by Garsia-Gessel in 1979.
