MATH 1951 Practice Exam 1

Name: _____

Instructions: This test should have 6 pages and 6 problems, and is out of 100 points. Please answer each question as completely as possible, and show all work unless otherwise indicated. You may use an approved calculator for this exam. (Approved: non-graphing, non-programmable, doesn't take derivatives)

1. For the function $f(x) = 3 + \frac{1}{x}$, find the function $f'(x)$ by using THE LIMIT DEFINITION OF THE DERIVATIVE.

2. For which values of a, b is the function

$$f(x) = \begin{cases} ax^2 - bx + 3 & \text{if } x < 2 \\ 3a + bx & \text{if } x = 2 \\ 5 & \text{if } x > 2 \end{cases}$$

continuous at $x = 2$?

3. You are roasting a turkey in an oven heated to 350 degrees Fahrenheit. Say that $T(x)$ represents the temperature of the turkey, in Fahrenheit, after x hours. You take some measurements and find that $T(2) = 300$ and $T'(2) = 20$.

(i) Write, in your own words, the meaning of the equation "$T(2) = 300$." This does not need to be a paragraph, but be as clear as possible.

(ii) Write, in your own words, the meaning of the equation "$T'(2) = 20$." This does not need to be a paragraph, but be as clear as possible. (HINT: thinking about the units of $T'(2)$ might help.)

(iii) Based on the information you have, roughly when do you think that the turkey will reach 310 degrees Fahrenheit?

4. Find the following limits. Each answer should be a number, ∞, $-\infty$, or DNE (for Does Not Exist).

(i) $\lim\limits_{t \to 6} \dfrac{(t-8)(t+5)}{4-t}$

(ii) $\lim\limits_{x \to -\infty} \dfrac{x^3 - 5x + 7}{2x^2 + 3}$

(iii) $\lim\limits_{x \to 1} \dfrac{\sqrt{x+3} - 2}{x-1}$

(iv) $\lim\limits_{h \to 0} \dfrac{1}{|h|}$

5. Find the derivatives of the following functions by using the differentiation rules we learned in class. YOU DO NOT NEED TO SIMPLIFY YOUR ANSWERS!!!

(i) $g(x) = \left(\sqrt{x} + \dfrac{1}{x^2}\right)e^x$

(ii) $r(x) = \left(\dfrac{x-2}{x+2}\right)^3$

(iii) $h(x) = \sin(\cos(\tan x))$

6. (i) Find all values a so that the tangent line at $x = a$ to the curve $y = \frac{x}{x+4}$ has slope 1.

(ii) For each value of a from part (i), find the equation of the tangent line to $y = \frac{x}{x+4}$ at $x = a$. Put your answers into slope-intercept form.