MATH-1951
Quiz 5 - (3.5.3.8)
Answer the following questions, and show your work. Answers need not be simplified.
Scientific calculator only.

[1] (5 points total)

a) (4 points) Find \( dy/dx \) by implicit differentiation.

\[ \sqrt{xy} + y = 1 \]

\[ \left( \frac{(xy)}{y} \right)' + y' = 0 \]
\[ \frac{1}{2} \frac{1}{(xy)^{1/2}} \left( x' \cdot y + x \cdot y' \right) + y' = 0 \]
\[ \frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}} \cdot y' + y' = 0 \]
\[ y' \left( \frac{x}{2\sqrt{xy}} + 1 \right) = \frac{-y}{2\sqrt{xy}} \]
\[ \left( \frac{x}{2\sqrt{xy}} + 1 \right) \]
\[ y' = \frac{-y}{\frac{x}{2\sqrt{xy}} + 1} \]

b) (1 point) Use your answer to part a) to find the slope of the tangent at \( x = 1/2, \ y = 1/2 \).

\[ y' \bigg|_{x=1/2, y=1/2} = \frac{-1}{2 \sqrt{\frac{1}{2} \cdot \frac{1}{2}} + 1} = \frac{-1}{\frac{1}{2} + 1} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3} \]
[2] (5 points total) The half-life of cesium-137 is 30 years. Suppose we have 100-mg sample.

a) (2.5 points) Find a formula for the mass remaining after $t$ years.

$$ p(t) = Ce^{kt} \quad p(30) = 50 \quad \text{by half-life} $$

$$ C = 100. $$

$$ 50 = 100e^{k \cdot 30} \quad \frac{1}{2} = e^{k \cdot 30} $$

$$ ln \frac{1}{2} = ln e^{k \cdot 30} \quad ln \frac{1}{2} = 30k \quad k = \frac{ln \frac{1}{2}}{30} = \frac{ln 1 - ln 2}{30} $$

$$ 50, p(t) = 100e^{-\frac{ln 2}{30} \cdot t} $$

b) (1 point) How much of the sample will remain after 100 years? Answer need not be simplified.

$$ p(100) = 100e^{-\frac{ln 2}{30} \cdot 100} = 100e^{\frac{112}{30}} \text{mg} $$

c) (1.5 points) After how long will only 1 mg remain? Answer need not be simplified.

Solve

$$ p(t) = 1 $$

$$ 1 = 100e^{-\frac{ln 2}{30} \cdot t} $$

$$ \frac{1}{100} = e^{-\frac{ln 2}{30} \cdot t} $$

$$ ln \frac{1}{100} = ln e^{-\frac{ln 2}{30} \cdot t} $$

$$ ln 100 = ln 1 - ln 100 = -\frac{ln 2}{30} \cdot t $$

$$ t = 30 \cdot \frac{ln 100}{ln 2} \text{ years} $$