MATH 1952 Practice Final Exam

Name: ____________________________

Instructions: This test should have 8 problems on 8 pages. Please answer each question as completely as possible, and show all work unless otherwise indicated. You may use an approved calculator for this exam. (Approved: non-graphing, non-programmable, doesn’t take derivatives) Answers do not need to be fully simplified unless otherwise specified in the problem.

1. At \( t = 0 \), a ball is thrown downwards at a speed of 32 feet per second from the lip of the Grand Canyon at a height of 1920 feet above the canyon floor.

   (a) Using the fact that acceleration is given by \( a(t) = -32 \) feet per second squared, find a formula for the velocity \( v(t) \) of the ball after \( t \) seconds. (Hint: what’s \( v(0) \)?)

   (b) Use your answer to (a) to give a formula for the height (or position) \( h(t) \) of the ball after \( t \) seconds. (Hint: what’s \( h(0) \)?) Use your answer to find out when the ball will hit the canyon floor.
2. Using the pictures below, give the values of the following definite integrals:

(a) $\int_{0}^{2} f(x) \, dx$

(b) $\int_{0}^{2} g(x) \, dx$

(c) $\int_{0}^{2} 2f(x) + 3g(x) \, dx$

(d) $\int_{2}^{4} f(x) \, dx$
3. Find the area between the curves $y = x^2$ and $y = 8 - x^2$. 
4. The curve \( y = \ln x \) between \( x = 1 \) and \( x = 3 \) is rotated around the \( x \)-axis, yielding a solid. Find the volume of that solid.
5. The curve \( x = \frac{y^2}{8} - \ln y \) between \( y = 1 \) and \( y = 3 \) is rotated around the \( x \)-axis, yielding a solid. Find the surface area of that solid.
6. Find the average value of the function \( f(x) = \frac{\sqrt{x^2-1}}{x^2} \) over the interval from \( x = 1 \) to \( x = \sqrt{2} \).
7. Approximate the definite integral \( \int_0^8 \frac{1}{1+x^2} \) by using

(a) Left endpoints with \( n = 4 \)

(b) Midpoints with \( n = 4 \)

(c) Simpson's Rule with \( n = 4 \).
8. Compute the following indefinite integrals via any method you wish.

(a) \( \int \frac{x^2+5}{\sqrt{x}} \, dx \)

(b) \( \int \frac{\sqrt{x}}{\sqrt{x}} \, dx \)

(c) \( \int \frac{4x+6}{x^2+4x+3} \, dx \)
Trig identities list:

\[ \sin^2 x + \cos^2 x = 1 \]
\[ \tan^2 x + 1 = \sec^2 x \]
\[ \cot^2 x + 1 = \csc^2 x \]
\[ \sin(2x) = 2 \sin x \cos x \]
\[ \cos(2x) = \cos^2 x - \sin^2 x \]
\[ \sin^2(x) = \frac{1 - \cos^2 x}{2} \]
\[ \cos^2(x) = \frac{1 + \cos^2 x}{2} \]