MATH-1952
Quiz 4 - (6.2)
Answer the following questions, and show your work. Scientific calculator only.

[1] (10 points total) Find the volume of the solids obtained by rotating the region in the first quadrant bounded by the curves $y = x$ and $y = x^3$ about the following lines.

(a) (4 pts) The $x$-axis.

\[ V = \pi \int_0^1 (x^2 - (x^3)^2) \, dx \]
\[ = \pi \int_0^1 x^2 - x^6 \, dx \]
\[ = \pi \left[ \frac{x^3}{3} - \frac{x^7}{7} \right]_0^1 \]
\[ = \pi \left( \frac{1}{3} - \frac{1}{7} \right) \]
\[ = \frac{4\pi}{21} \]

(b) (6 pts) The $y$-axis. (Hint: You’ll need to use $x = y^{1/3}$ instead of $y = x^3$.)

\[ V = \pi \int_0^1 (y^2 - (y^{1/3})^2) \, dy \]
\[ = \pi \int_0^1 x^{2/3} - x^2 \, dx \]
\[ = \pi \left[ \frac{3}{5} y^{5/3} - \frac{x^3}{3} \right]_0^1 \]
\[ = \pi \left( \frac{3}{5} \cdot \frac{1}{3} - (0) \right) \]
\[ = \frac{4\pi}{15} \]