MATH-1952
Quiz 5 - (6.3, 6.4)
Answer the following questions, and show your work. Scientific calculator only.

[1] (6 points) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = 4x - x^2$ and $y = x$ about the $y$-axis.

Intersection points: $x = 1$, $4 - 1^2 = 3$; $x = 3$.

Volume:

$$V = 2\pi \int_0^3 2x - x^2 - x \, dx$$

$$= 2\pi \int_0^3 (2x - x^2) \, dx$$

$$= 2\pi \left[ x^2 - \frac{x^3}{3} \right]_0^3$$

$$= 2\pi \left( 9 - \frac{27}{3} \right)$$

$$= 2\pi \left( 9 - 9 \right)$$

$$= 0$$

[2] (4 points) A variable force of $8x^{-3}$ pounds acts on an object when it is $x$ feet from the origin. Calculate the work done in moving the object from $x = 1$ ft to $x = 2$ ft.

$$W = \int_1^2 8x^{-3} \, dx$$

$$= 8 \left[ -\frac{x^{-2}}{-2} \right]_1^2$$

$$= 8 \left[ -\frac{1}{x^2} \right]_1^2$$

$$= 8 \left( -\frac{1}{4} - \frac{1}{1} \right)$$

$$= -1 + 4 = 3$$