MATH 1953 Final Topics List/Study Guide

Here is a list of topics that are fair game for the exam.

• 10.1: Know what a parametrically defined function is and how to graph one, either via table of values or by eliminating the parameter.

• 10.2: Know how to find dy/dx for parametric curves and use this to solve associated tangent line problems.

• 10.2: Know how to find the area between a parametric curve and the x-axis.

• 10.2: Know how to find the arclength of a parametric curve.

• 10.3: Know what polar coordinates are, and how to graph a function defined by a polar equation.

• 10.3: Know how to find dy/dx for polar curves and use this to solve associated tangent line problems.

• 10.4: Know how to find area for polar curves; this includes both "simple" area problems, where you just find the area "swept out" by line segments from the origin to the curve, and more complicated area problems, for instance where you must find the area inside one curve and outside another.

• 10.4: Know how to find arclength for polar curves.

• NOTE: For the last two bullet points, you also need to be able to figure out on your own what range of θ determines one complete tracing of the graph if the graph is a loop shape.

• 4.4: Know how to apply L'Hospital's rule to find limits of certain fractions, products, differences, sums, and exponentials.

• 7.8: Know what an improper integral represents (both types), know how to evaluate them, know meaning of terms "converge" and "diverge."

• 7.8: Know how to use the Comparison Test to either show that an improper integral of a positive function converges (by showing that it is less than a function whose integral converges) or diverges (by showing that it is greater than a function whose integral diverges.)

• 11.1-11.2: Know what sequences and series are; know what it means for a sequence and/or series to converge or diverge.

• 11.2: Know how to find the exact sum of geometric series and telescoping series.

• 11.2: Know the values of p for which a p-series $\left(\sum_{n=1}^{\infty} \frac{1}{n^p}\right)$ converges and diverges.

• 11.3-11.7: Know all of the tests discussed in class to check convergence or divergence of a series. This includes being able to decide which test to use for a given series, and being able to verify the hypotheses for your chosen test. The tests you should know are the Divergence Test (11.1), the Integral Test (11.3), the Comparison and Limit Comparison Tests (11.4), the Alternating Series Test (11.5), and the Ratio and Root Tests (11.6). (see the convergence test summary I provided for the 2nd midterm)

• 11.5: Know how to approximate an alternating series to any desired tolerance by taking partial sums and using the inequality $|R_n| \leq |a_{n+1}|$, where a_{n+1} is the $(n+1)$th term and $R_n = L - s_n$ is the error between the sum of the series and the nth partial sum. (Another way to state this more simply is that you find the first term of the alternating series whose absolute value is less than your desired error, and cut the series off before that term.)

• 11.6: Know the definition of absolute convergence, know how to check absolute convergence of a series, and know that absolute convergence of a series implies usual convergence.

• 11.8: Know the definition of a power series centered at a number a, how to use the Root (or Ratio) test to find the radius of convergence of a power series, and how to find the interval of convergence of a power series by checking endpoints.

• 11.9: Know how to manipulate known power series to get new ones. Techniques we know are multiplication by a constant/polynomial, addition or subtraction of two power series, termwise differentiation, termwise integration, and substitution. Also, know how the radius and interval of convergence are affected by each of these operations.

• 11.10 Know what a Taylor series of a function $f(x)$ centered at a number a is, and know the formula $a_n = \frac{f^{(n)}(a)}{n!}$ for the coefficient of $(x-a)^n$ in the series. Know the Taylor series for the basic functions we've repeatedly discussed in class ($\sin x$ at $a=0$, $\cos x$ at $a=0$, e^x at $a=0$, $\tan^{-1}(x)$ at $a=0$, $\frac{1}{1-x}$ at $a=0$.)

• 11.10 Know how to use ideas from 11.9 and 11.10 to find Taylor series for complicated functions in an efficient way. (For instance, to find the Taylor series for $f(x) = x\tan^{-1}(x^2)$ at $a=0$, the easiest way is to begin with the Taylor series for $\tan^{-1}(x)$, substitute x^2 for x, then multiply by x.)

• 11.11 Know how to use alternating series approximations to compute approximations to a function to any desired tolerance by taking partial sums of its Taylor series.