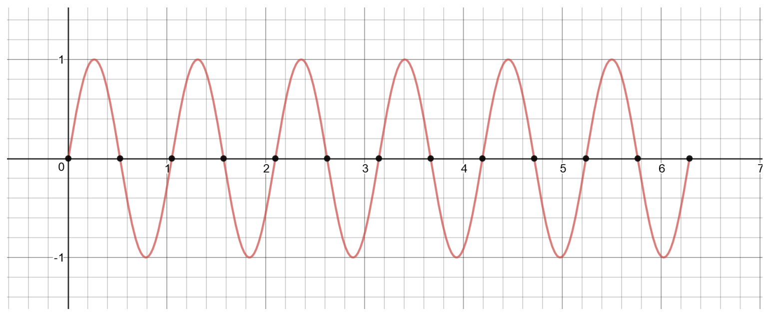
Homework 2 - Solutions

Calculus III

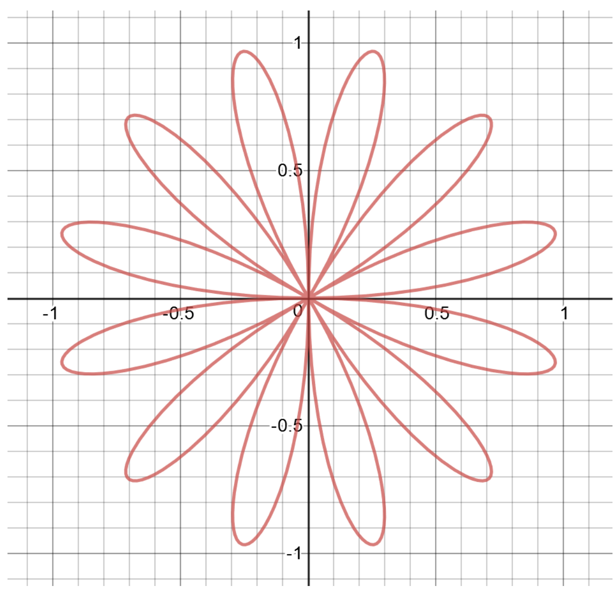
Exercise 1

Solution.

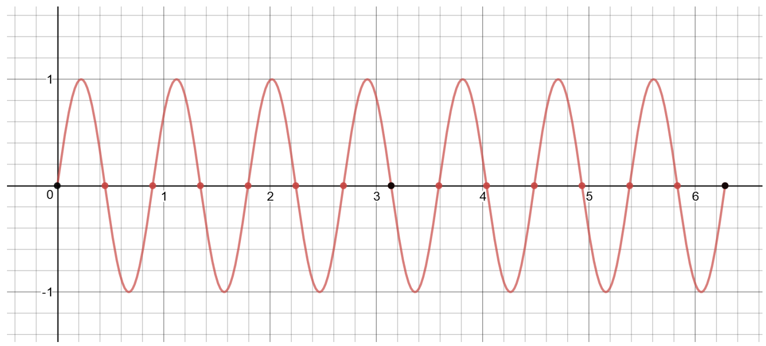
(a) On $[0, 2\pi]$, the graph of $y = \sin(6\theta)$ has 12 subintervals on which $\sin(6\theta)$ is positive or negative.



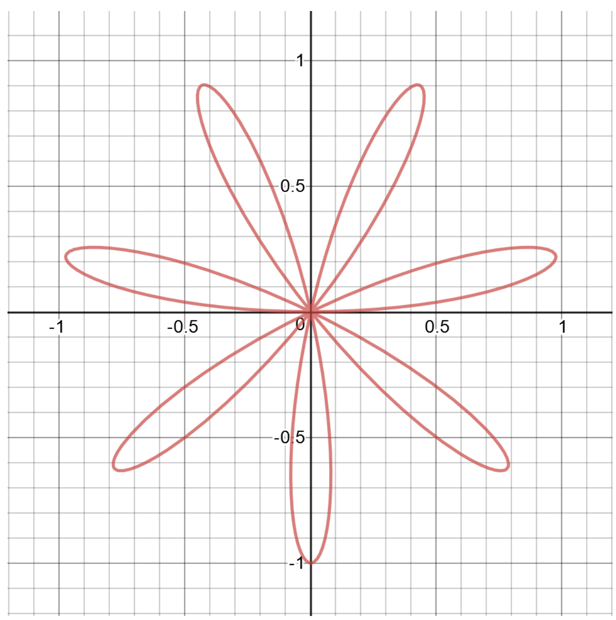
The zeros of $r = \sin(6\theta)$ correspond to points at the origin, hence each of the 12 subintervals forms a petal. The graph of $r = \sin(6\theta)$ on $[0, 2\pi]$ is a flower with 12 petals:



(b) The graph of $y = \sin(7\theta)$ is broken up into 14 subintervals:



However, here we notice that for $\theta \in [0, \pi]$, $\sin(7(\pi + \theta))$ is equal to $-\sin(7\theta)$. Since for any radius r and angle θ the points (r, θ) and $(-r, \pi + \theta)$ are equal (a fact of polar coordinates), then the graph of $r = \sin(7\theta)$ on $[\pi, 2\pi]$ retraces the graph of $r = \sin(7\theta)$ on $[0, \pi]$. The graph of $r = \sin(7\theta)$ on $[0, 2\pi]$ is a flower with 7 petals:



(c) Assuming $n > 0$, the area of a single petal of the graph of $r = \sin(n\theta)$ is given by

$$\begin{aligned}
A &= \frac{1}{2}\int_0^{\frac{\pi}{n}} \sin^2(n\theta)\, d\theta \\
&= \frac{1}{2}\int_0^{\frac{\pi}{n}} \frac{1-\cos(2n\theta)}{2}\, d\theta \\
&= \frac{1}{4}\int_0^{\frac{\pi}{n}} (1-\cos(2n\theta))\, d\theta \\
&= \frac{1}{4}\left[\theta - \frac{1}{2n}\sin(2n\theta)\right]_0^{\frac{\pi}{n}} \\
&= \frac{1}{4}\left[\left(\frac{\pi}{n} - 0\right) - (0-0)\right] \\
&= \frac{\pi}{4n}
\end{aligned}$$

If n is even, then the flower has $2n$ petals and a total area of $2n \cdot \frac{\pi}{4n} = \frac{\pi}{2}$. If n is odd, then the flower has n petals and a total area of $n \cdot \frac{\pi}{4n} = \frac{\pi}{4}$.

Exercise 2

Solution. The curve $r = C + \sin(6\theta)$ will give a flower as long as $|C| \leq 1$. This is because petals are formed on subintervals whose endpoints evaluate to 0. If $|C| > 1$ then $C + \sin(6\theta)$ has no zeros.