Summary of convergence/divergence tests for infinite series:

- **Geometric series:** The series $\sum_{n=0}^{\infty} r^n$ converges to $\frac{1}{1-r}$ if $-1 < r < 1$ and diverges otherwise.

- **p-series:** The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise.

- **Divergence Test:** If the terms x_n do not approach 0 as $n \to \infty$, then the series $\sum_{n=1}^{\infty} x_n$ **diverges**.

- **Integral Test:** If the terms x_n are positive and decreasing, and x_n can be "turned into a function" $f(x)$, then the series $\sum_{n=1}^{\infty} x_n$ and the improper integral $\int_{n=1}^{\infty} f(x)\, dx$ **have the same convergence status**.

- **Comparison Test:** If $0 \leq x_n \leq y_n$ for all n, and the new series $\sum_{n=1}^{\infty} y_n$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ **converges**. If $0 \leq y_n \leq x_n$ for all n, and the new series $\sum_{n=1}^{\infty} y_n$ diverges, then the original series $\sum_{n=1}^{\infty} x_n$ **diverges**. If you get a situation/inequality not listed above, the test was **inconclusive** and you have to use another one.

- **Limit Comparison Test:** If $\frac{x_n}{y_n} \to L \neq 0$ (and L is a finite number), then the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ **have the same convergence status**. If $L = 0$ or $L = \pm\infty$, the test was **inconclusive** and you have to use another one.

- **Alternating Series Test:** If you have a series $\sum_{n=1}^{\infty} (-1)^n x_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} x_n$, and the terms x_n are positive, approach 0, and are decreasing, then the original series **converges**.

- **Absolute Convergence Test:** If you have a series $\sum_{n=1}^{\infty} x_n$, and the series $\sum_{n=1}^{\infty} |x_n|$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ **absolutely converges**, which means that it **converges**.

- **Root Test:** If $\sqrt[n]{|x_n|} = |x_n|^{1/n}$ approaches a limit R, then the series $\sum_{n=1}^{\infty} x_n$ **converges** if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ **diverges** if $R > 1$. If $R = 1$, this test was **inconclusive** and you have to use another one.

- **Ratio Test:** If $\frac{|x_{n+1}|}{|x_n|} = \left|\frac{x_{n+1}}{x_n}\right|$ approaches a limit R, then the series $\sum_{n=1}^{\infty} x_n$ **converges** if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ **diverges** if $R > 1$. If $R = 1$, this test was **inconclusive** and you have to use another one.

Approximation formulas:

- **Positive series:** For an infinite series $\sum_{n=1}^{\infty} f(n)$ with positive and decreasing terms, the error from approximating with the Nth partial sum is less than $\int_{N}^{\infty} f(x)\, dx$.

- **Alternating series:** For an infinite series $\sum_{n=1}^{\infty}(-1)^n x_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} x_n$ with x_n positive and decreasing, the error from approximating with the Nth partial sum is less than x_{N+1}.

Formulas for parametric/polar functions:

- **Parametric slope:** $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$ or $\frac{y'}{x'}$

- **Parametric area:** Area $= \int_{t_1}^{t_2} y \frac{dx}{dt}\, dt$ or $\int_{t_1}^{t_2} yx'\, dt$

- **Parametric arc length:** Length $= \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ or $\int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2}\, dt$

- **Polar conversion:** $y = r\sin\theta$, $x = r\cos\theta$, $r = \sqrt{x^2 + y^2}$, $\tan\theta = \frac{y}{x}$

- **Polar slope:** $\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$ or $\frac{(r\sin\theta)'}{(r\cos\theta)'}$

- **Polar area:** Area $= \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2\, d\theta$

- **Polar arc length:** Length $= \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

- **Useful trig formulas:** $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$, $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

Taylor series:

- **Taylor series formula:** The Taylor series for $f(x)$ centered at $x = a$ is $\sum_{n=0}^{\infty} c_n (x-a)^n$, where $c_n = \frac{f^{(n)}(a)}{n!}$.
  - $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, interval of convergence $(-\infty, \infty)$
  - $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, interval of convergence $(-\infty, \infty)$
  - $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$, interval of convergence $[-1, 1]$
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, interval of convergence $(-\infty, \infty)$
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, interval of convergence $(-1, 1)$