MATH 3162 Homework Assignment 1

Instructions: Solve and turn in all of the assigned problems, showing ALL steps or reasoning used in your solutions.

Due on Monday, January 14th, at the BEGINNING of class.

- Abbott: 4.4.1(a,b,c), 4.4.5

- The set $S = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ is not compact. Prove that it is not compact in two ways, by proving the negations of the sequential and open cover definitions of compactness respectively. (Phrased a different way: do problems 3.3.2, 3.3.11 from Abbott for this set S.)

- Prove that if $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ are functions which are each uniformly continuous on D, then $f + g : D \to \mathbb{R}$ is uniformly continuous on D as well.

- If $f : \mathbb{R} \to \mathbb{R}$ and $x_0 \in \mathbb{R}$ have the property that $\exists \delta > 0$ s.t. $\forall \epsilon > 0$, $|x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon$, prove that f is constant on the interval $(x_0 - \delta, x_0 + \delta)$.

Extra problems for graduate students:

- Abbott: 4.4.2(c), 4.4.6(a,b) (give proofs here, not "short explanations" as asked for in the problem), 4.4.11