MATH 3162 Homework Assignment 2

Instructions: Solve and turn in all of the assigned problems, showing ALL steps or reasoning used in your solutions.

Due on Monday, January 21st, at the BEGINNING of class.

Abbott: 4.4.13(a), 4.5.2(a,b,c), 4.5.3, 5.2.6(a)

• If f is continuous and 1-1 on $[a, b]$, prove that f is either strictly increasing or strictly decreasing on $[a, b]$. (f is strictly increasing on $[a, b]$ if $\forall x, y$, $x < y \Longrightarrow f(x) < f(y)$, and is strictly decreasing on $[a, b]$ if $\forall x, y$, $x < y \Longrightarrow f(x) > f(y)$.)

• If f is continuous on $[a, b]$, $f(a) < 0 < f(b)$, $S = \{x \in [a, b] \ : \ f(x) \leq 0\}$, and $c = \sup(S)$, prove that $f(c)$ is NOT negative.

• If $f : \mathbb{R} \to \mathbb{R}$, f is differentiable at $x = c$, and $f'(c) > 0$, prove that $f(c)$ is not a maximum or minimum value of f. ($f(c)$ is a maximum value of f if it is greater than or equal to $f(x)$ for all $x \in \mathbb{R}$, and a minimum value of f is defined similarly.)

• Suppose that f is differentiable on $[a, b]$ and that for all $x \in [a, b]$, $f'(x) \neq 0$.
(a) Prove that either $f'(x) > 0$ for all $x \in [a, b]$ or $f'(x) < 0$ for all $x \in [a, b]$.
(b) Prove that either f is strictly increasing on $[a, b]$ or f is strictly decreasing on $[a, b]$. (Hint: Mean Value Theorem)

Extra problems for graduate students:

Abbott: 4.4.13(b), 4.5.8, 5.2.6(b)