Topics list for Math 3162 final exam

The midterm will be held in class on Thursday, March 21st. Calculators, laptops, notes, cheat sheets, etc. will NOT be allowed. The midterm will cover the material from Sections 4.4-4.5, 5.2-5.4, 6.2-6.7, and 7.2-7.6 of the textbook (not including Abel's Test).

The best ways to prepare for the midterm are to review your class notes, re-read the text material, review the homework problems, and work on other problems in the text. Below is a review of some the main topics from the course.

4.4: Know properties of a continuous function f with compact domain K, namely that f is uniformly continuous on K and that the range $f(K)$ is compact (implying that f achieves a max and min on K).

4.5: Know the Intermediate Value Theorem and examples of its application.

5.2: Know the definition of the derivative and its relationship with difference quotients. Know how to prove basic theorems using this definition (for instance, that $(f+g)'(x) = f'(x)+g'(x)$.) Know how to use information about difference quotients to derive information about a derivative (e.g. the Interior Extremum Theorem.) Know Darboux's theorem and how it can be applied to the derivative even though the derivative may not be continuous (e.g. if $f'(x)$ is never 0, then $f'(x)$ is either always positive or always negative.)

5.3: Know the Mean Value Theorem and Generalized Mean Value Theorem, and how they let you use information about the derivative to yield information about difference quotients (e.g. if $f'(x) > 0$ on (a, b), then f is strictly increasing on (a, b).) Know the 0/0 case of L'Hospital's Rule, and how it can help you compute some limits.

5.4: There's not much testable in this section, but know that this example exists, its properties, and (most importantly) how its continuity is derived from results about uniformly convergent series in Chapter 6.

6.2: Know the definition of uniform convergence, and how it differs from pointwise convergence. Know how to verify uniform convergence using the definition, or disprove it using the negation. Know the Cauchy criterion for uniform convergence. Know that the uniform limit of continuous functions is continuous.

6.3: Know how uniform convergence relates to differentiability, and how to use uniform convergence of f_n' to prove differentiability of the limit of f_n for a sequence (f_n) of functions.

6.4: Know how to apply the results of 6.2 and 6.3 to a series $\sum_{k=0}^{\infty} f_k(x)$ of functions, by considering (uniform) convergence of the sequence of partial sums $S_n = \sum_{k=0}^{n} f_k(x)$. Know the Term-by-Term Differentiability Theorem and the Weierstrass M-Test.

6.5: Know the definition of a power series, and definitions of the interval of convergence and radius of convergence R. Know that R can be written as $\dfrac{1}{\limsup \sqrt[n]{|a_n|}}$ (with the conventions that $\frac{1}{0} = \infty$ and $\frac{1}{\infty} = 0$). Know where you can guarantee uniform convergence of a power series and thereby use results from 6.4 to verify continuity.

6.6: Know how to prove that a Taylor series converges at a value of x by using Lagrange's Remainder Theorem. Know how to then prove uniform convergence on an interval using previous results from Section 6.5.

6.7: Know the Weierstrass Approximation Theorem.

7.2: Know the definition of the Riemann integral, and how it relates to upper/lower sums for partitions. Know how finite telescoping sums can appear in calculations of upper/lower sums (for instance, the commonly used fact that $\displaystyle\sum_{k=0}^{n-1}(x_{k+1} - x_k) = b - a$ for any partition P.)

7.3: Know that functions with finitely many discontinuities are integrable, and that there exist bounded functions (like the Dirichlet function) which are not integrable.

7.4: Know basic rules regarding how integration behaves under basic operations, like $\int_a^b f+g\ dx = \int_a^b f\ dx + \int_a^b f\ dx$. Know that most proofs of integration/value of integrals involve finding a partition P where $U(f,P) - L(f,P)$ is small, and that once you prove integrability, $\int_a^b f\ dx$ is trapped between $L(f,P)$ and $U(f,P)$ for every partition P. Know how convergence relates to integration, namely that pointwise convergence of a sequence f_n of integrable functions almost never implies anything about integrability of the limit, but uniform convergence of f_n to f implies that f is integrable and that $\int_a^b f_n\ dx \to \int_a^b f\ dx$.

7.5: Know the Fundamental Theorem of Calculus and how it relates derivatives and integrals. Know how to evaluate basic integrals on finite intervals by using an antiderivative and applying the FTC (e.g. $\int_5^6 x^2\ dx = \frac{6^3}{3} - \frac{5^3}{3}$ since $\left(\frac{x^3}{3}\right)' = x^2$). Know that if a Taylor series converges uniformly on $[a,b]$, you can use results from 7.4 along with the FTC to write a formula for seemingly impossible integrals, such as $\int_0^1 e^{-x^2/2}\ dx =$

$$\int_0^1 \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(n!2^n)} x^{2n} \right) dx = \sum_{n=0}^{\infty} \int_0^1 \frac{(-1)^n}{(n!2^n)} x^{2n}\ dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)2^n}.$$

7.6: Know the definition of measure zero, and Lebesgue's criterion for integrability, which states that a function f bounded on $[a,b]$ is integrable if and only if its set of discontinuities has measure zero.