MATH 3162 Practice Midterm Exam

Instructions: You may not use any instructional aids (book, notes, calculator, etc.) on this exam. For problems which require proof, you should prove everything either from basic building blocks of the real numbers or clearly stated facts proved on the homework or in class (such as the fact that the union of two countable sets is countable). Pay attention to particular facts that I do or do not allow you to use for particular problems! You may use the back pages as scratch paper, but make sure that your answers are clearly indicated.

Name: ____

1. For each of the following, either provide an example with the desired properties or explain (via a theorem or fact that we proved in class) why such an example cannot exist. (You DO NOT need to provide a proof that your example has the desired properties!)

(a) A power series $\sum_{n=0}^{\infty} a_n x^n$ which converges at $x = 2$ and diverges at $x = -3$

(b) A power series $\sum_{n=0}^{\infty} a_n x^n$ which converges at $x = -3$ and diverges at $x = 2$

(c) A function f where f is differentiable on $\mathbb{R}$ but f is not continuous on $\mathbb{R}$

(d) A function f where f is differentiable on $\mathbb{R}$ but f' is not continuous on $\mathbb{R}$

2. **(a)** State the definition of "$f(x)$ is uniformly continuous on the set A."

(b) Use your answer to (a) to show that $f(x) = x^3$ is uniformly continuous on $[1, 3]$.

(c) Use your answer to (a) to show that $f(x) = x^3$ is not uniformly continuous on $\mathbb{R}$.

3. (a) State the definition of "The sequence $f_n(x)$ uniformly converges to $f(x)$ on the set A."

(b) Use your answer to (a) to prove that if each $f_n(x)$ is uniformly continuous on a set A, and $f_n(x)$ converges uniformly to $f(x)$ on A, then $f(x)$ is uniformly continuous on A.

4. (a) For a differentiable function $f(x)$, give the definition of $f'(c)$ at a value $x = c$.

(b) Use your answer to (a) to prove the following: if f is differentiable on $\mathbb{R}$ and $\exists c$ s.t. $f(c)$ is a global maximum of f (i.e $\forall x \in \mathbb{R}$, $f(x) \leq f(c)$), then $f'(c) = 0$.

5. (a) State the Intermediate Value Theorem.

(b) Use your answer to (a) to show that f is continuous on a closed interval $[a, b]$, then $f([a, b])$ is also a closed interval.

6. (a) State the Mean Value Theorem.

(b) Use your answer to (a) to prove that if $f(x)$ is differentiable on $[0, 3]$, $f(0) = 5$, and $f'(x) > 3$ for all $x \in (0, 3)$, then $f(x) > 5 + 3x$ for all $x \in [0, 3]$.