MATH 3851 Homework Assignment 2 (due Tuesday, January 23rd)

Textbook problems:

Section 12, problem 5: If we define the set $S = \{z : |z| < 1\} \cup \{z : |z-2| < 1\}$, explain (you don't need to give a formal proof) why S is NOT connected.

Section 14, problem 2(a): Write the function $f(z) = z^3 + z + 1$ in the form $f(z) = u(x, y) + iv(x, y)$. (Here, $z = x + iy$.)

Section 14, problem 4: Write the function $f(z) = z + \frac{1}{z}$ in the form $f(z) = u(r, \theta) + iv(r, \theta)$. (Here, $z = re^{i\theta}$.)

Section 18, problem 1(b): Use the ϵ-δ definition of limit to prove that $\lim_{z \to z_0} \overline{z} = \overline{z_0}$ for any $z_0 \in \mathbb{C}$. (It may be easier to start with the easier limit question in the "Extra Problems.")

Section 18, problem 5: Show that the limit of the function $f(z) = \left(\frac{z}{\overline{z}}\right)^2$ as z approaches 0 does not exist. Hint: consider points of the form $z = x$ and $z = x + xi$ approaching the origin.

Extra problems:

• For the function $f(z) = 5z - i$, show using the ϵ-δ definition of limit that $\lim_{z \to 3} f(z) = 15 - i$.

• Prove that $\overline{e^z} = e^{\overline{z}}$ for any complex number z.

• For each of the following sets, say whether or not they are (i) open, (ii) closed, (iii) connected, (iv) bounded. If you claim the set does not have some of these properties, explain why. (For instance, if you're saying the set isn't open, give an example of an interior point which isn't in the set. If you're saying that the set isn't bounded, explain why it doesn't lie inside any circle.)

(a) $\{z : |2z + 3| > 4\}$
(b) $\{z : \text{Im(z)} = 1\}$
(c) $\{z : 0 \leq \arg(z) < \frac{\pi}{4}\}$

• Suppose that U and V are connected sets, and that the intersection $U \cap V$ is nonempty. Explain why $U \cup V$ must be connected.