MATH 3851 Homework Assignment 8 (does NOT need to be turned in; just for practicing week 10 concepts)

Textbook problems:

Section 57: (p. 170-172) 10 Section 59: (p. 177-178) 1

Extra problems:

• Do problem 2(b) from Section 52 (page 170) for the contour C which is the circle $\{z : |z| = 5\}$, traversed in the CLOCKWISE direction.

• For all of the following, $f(z) = \frac{1}{z^2+1}$, L_R is the line segment from -R to R, C_R is the top half of the circle with radius R centered at 0 and oriented counterclockwise, and Γ_R is the loop $L_R + C_R$.

(a) Show that for every R > 1, $\int_{\Gamma_R} f(z) dz = \pi$. (b) Show that $\lim_{R\to\infty} \int_{C_R} f(z) = 0$ by using the LR-bound and the triangle inequality (as we've done in class a couple of times.) (c) Use (a) and (b) to show that $\int_{-\infty}^{\infty} f(x) dx = \lim_{R\to\infty} \int_{L_R} f(z) dz = \pi$.