**Practice Final Exam for Math 3851**

**1.** (10 min.) Write all cube roots of $-8$, in both rectangular ($z = x + iy$) and polar/exponential ($z = re^{i\theta}$) form.

**2.** (6 min.) Suppose that $f = u + iv$ is an entire function, and that $u = v$ everywhere in $\mathbb{C}$. Explain why $f$ must be a constant.

**3.** (6 min.) Prove the identity $\cos(z - \frac{\pi}{2}) = \sin(z)$ by using the definitions of $\sin z$ and $\cos z$ in the complex plane.

**4.** (8 min.) Find all values of $z$ where the function $f(z) = \overline{z}^2$ is differentiable, and find all values where it is analytic.

**5.** (18 min.) Compute the integral $\int_\Gamma \overline{z}^2 \, dz$, where $\Gamma$ is the square with vertices 0, 1, $1 + i$, and $i$, traversed counterclockwise.

**6.** **(a)** (2 min.) For the function $f(z) = \frac{z^2+1}{z(z^2-1)}$, what are the values of $z$ where $f$ is NOT analytic?

**(b)** (10 min.) For each point of non-analyticity $z_0$ in part (a), find the value of $\int_C f(z) \, dz$, where $C$ is a very small circle centered at $z_0$. (You should get one answer here for each of your answers from (a).)

**(c)** (3 min.) Find a closed simple contour $\Gamma$ for which $\int_\Gamma f(z) \, dz = 4\pi i$. (Hint: can you combine your integrals from (b) to get to $4\pi i$ somehow?)

**7.** **(a)** (5 min.) Find a branch of $\log(iz - 1)$ which is analytic in the domain $D = \{z \; : \; \mathrm{Im}(z) > 0\}$.

**(b)** (6 min.) Use your answer to part (a) to find $\int_\Gamma \frac{1}{iz-1} \, dz$, where $\Gamma$ is a contour consisting of a line segment from $1 + i$ to $-2 + 3i$, followed by a line segment from $-2 + 3i$ to $4i$, followed by a line segment from $4i$ to $-1 + i$. (HINT: you should be able to do this WITHOUT parametrizing this contour!)

**8.** (8 min.) Find the contour integral $\int_C \left(\frac{z-i}{z+i}\right)^3 \, dz$, where $C$ is the circle of radius 2 centered at the origin, traversed clockwise.

**9.** (8 min.) Use the LM bound to give any upper bound for $\left| \int_\Gamma \frac{z^2+1}{z^4-2} \, dz \right|$, where $\Gamma$ is the circle of radius 3 centered at the origin, traversed in either direction. You DO NOT need to compute the integral itself!

**10.** (5 min.) Explain why the function $f(z) = f(x + iy) = e^{\sin(x^{10}) + i\cos(y^{10})}$ is not entire. (HINT: think about the modulus of $f$!)