Practice Midterm for Math 3851

1. Write $(-1 + \sqrt{3}i)^{10}$ in polar form (i.e. $re^{i\theta}$) in at least two different ways.

2. Write all possible values of $(1 + i)^{1+i}$ in rectangular form (i.e. $x + iy$.)

3. At which $z \in \mathbb{C}$ is the function $f(z) = f(x+iy) = 2x^3y - 2xy^3 + i(x^4 + y^4)$ differentiable? At which values is it analytic?

4. Find a function v so that $f(z) = f(x + iy) = (2x^3y - 3xy^3) + iv(x,y)$ is entire, i.e. analytic throughout $\mathbb{C}$.

5. Prove the following statement: if $\left|\frac{i-z}{i+z}\right| = 1$, then z is real.

6. Find a branch of the multiple-valued expression $f(z) = \log(i(z + 2i))$ which is analytic for all z in the open disk $D = \{z : |z| < 2\}$.

7. Find all values of $z \in \mathbb{C}$ for which $\cos z = 2$, and write your answer(s) in rectangular form. (Hint: You can reduce $\cos z = 2$ to a quadratic equation with variable e^{iz}.)

8. Use the triangle inequality to justify the following inequalities: $\frac{12}{5} \leq |\sin(\ln 3 + i\ln 5)| \leq \frac{13}{5}$. (Hint: Look at the moduli of e^{iz} and e^{-iz}.)