MATH 4290 Homework Assignment 5

Due on Thursday, October 25th, at the BEGINNING of class.

• If (X, T) is a topological dynamical system and X is a compact metric space with metric d, prove that the quantity d_n defined by $d_n(x, y) := \max_{0 \leq i < n} d(T^i x, T^i y)$ is a metric for all $n \in \mathbb{N}$.

• Prove the inequalities in Lemma 2.5.1 of the textbook.

• For the one-sided full shift $(\{0, 1\}^{\mathbb{N}}, \sigma)$, find, with proof, $\operatorname{sep}(n, 2^{-k})$ and $\operatorname{span}(n, 2^{-k})$ for every $n \in \mathbb{N}$ and $k \in \mathbb{N} \cup \{0\}$. (Reminder: the metric here is $d((x_n), (y_n)) := 2^{-\max\{n \geq 0 \,:\, x_i = y_i \ \forall i \leq n\}}$.)

• If (X, T) factors onto (Y, S), prove that $h(X, T) \geq h(Y, S)$.

• If (X, T) is an isometry (i.e. $d(x, y) = d(Tx, Ty)$ for all $x, y \in X$), prove that $h(X, T) = 0$.