MATH 1150 Homework Assignment 2

1. (warmup) A casino offers the following game: you choose a card at random from a standard deck of 52 cards. If the card is an Ace, you win \$10. If the card is a face card (Jack, Queen, or King), you win \$5. Otherwise, you lose \$3. What is the expected value of this game for you? In the long run, is it a good idea to play this game?

2. Rose and Colin play the following game: Rose picks either 1 or 4 and Colin picks either 2 or 3. If the numbers picked are consecutive, then Colin pays Rose a payoff equal to the product of the numbers, and if they are not, then Rose pays Colin a payoff equal to the product of the numbers. (Consecutive just means that the larger number is one bigger than the smaller: for instance, 2 and 1 are consecutive, but 2 and 4 are not.)

(a) Write the matrix of payoffs to Rose for this game.

(b) As we did in class on Tuesday, set x to be the probability that Rose plays strategy 1 (then $1 - x$ is the probability that she plays strategy 4). Then, graph the expected payoffs for each of Colin's counterstrategies (represented by the y-coordinate) versus Rose's mixed strategy choice (represented by the x-coordinate). Your final graph should have two lines, one representing Colin A and one representing Colin B, extending from $x = 0$ to $x = 1$. (If you missed class, take a look at Figure 3.1 on p. 16 of Straffin, it's the same basic idea.) What mixed strategy should Rose play in this game?

(c) Now draw the corresponding graph where the roles of Rose and Colin are switched, and determine what mixed strategy Colin should play in this game.

(d) What is the value of this game?

3. Consider the game with the following matrix of payoffs to Rose:

	A	B
A	1	-1
B	2	0
C	-1	3

(a) Does this game have a saddle point?

(b) Suppose Rose is thinking of playing the following mixed strategy: A with 50% chance, B with 30% chance, and C with 20% chance. Would you advise her against this? Can you come up with an alternate mixed strategy which should be better? (HINT: are any of Rose's strategies dominated? Maybe she should replace those with something better!) Verify that your new mixed strategy "dominates" Rose's old strategy by showing that for any strategy Colin picks, your new strategy gives a larger expected payoff for Rose.