# MATH 1150 Homework Assignment ~~4~~ 5 Solutions

**Show your work!!!**

**1.** Interpret some real-world example (this can either be autobiographical or fictional) as a non-zero sum game. In other words, take some situtation involving two people's interests, where each player has a finite number of options (strategies) available to them, and write a payoff matrix where you give numerical values to each outcome. Explain as well as you can your motivation for choosing the matrix that you did. Also, draw the movement diagram for your example, and tell me which of the locations in your payoff matrix are Nash equilibria or Pareto optimal outcomes.

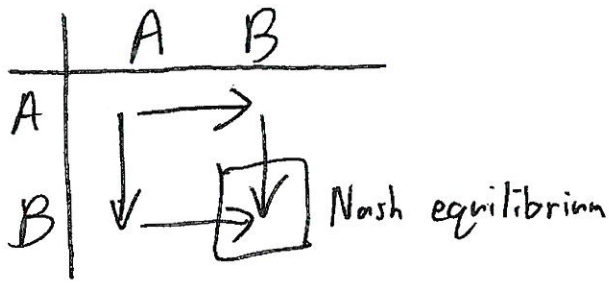
(If you're confused as to what I'm asking, the games of Chicken or Prisoner's Dilemma that we discussed in class are excellent examples to start from.)

**Solution:** Obviously there were many correct answers for this question. Here is one of my favorite submissions: (slightly paraphrased)

Two friends are putting their hands up for a high-five. Each player has to choose whether to go for the high-five or pull their hand away. If both players go for the high-five, they get payoffs of 2 because they get got an awesome high-five and some satisfaction. If one player goes for the high-five and the other takes their hand away, the one who took their hand away gets a payoff of 3 because there is a lot of satisfaction from making the other player look dumb, and the one who went for the high-five gets a payoff of 0 because they look dumb. If both players pull their hands away, they each get a payoff of 1 because they both feel a little silly. The payoff matrix is then

| | A | B |
|---|---|---|
| A | (2,2) | (0,3) |
| B | (3,0) | (1,1) |

Movement diagram

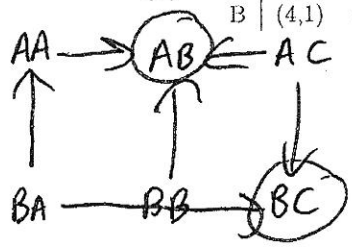



There is only one Nash equilibrium of $(1, 1)$ at BB, which is not Pareto optimal since the outcome of $(2, 2)$ at AA is preferable for both players.

(b)

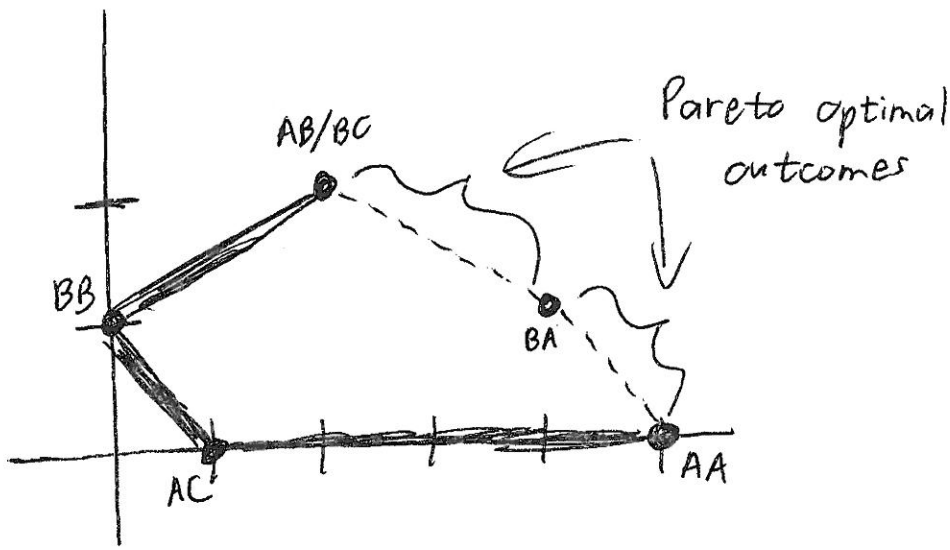
| | A | B | C |
|---|---|---|---|
| A | (5,0) | (2,2) | (1,0) |
| B | (4,1) | (0,1) | (2,2) |

(i)




equilibria at AB, BC

(ii)




(iii) The game is NOT solvable in the strict sense; though the two Nash equilibria are both Pareto optimal and have the same payoffs for both players, it is possible for both players to attempt to hit one and arrive at a worse outcome. For instance, Rose could play A attempting to get to the Nash equilibrium at AB, and Colin could play C attempting to get to the Nash equilibrium at BC, and they would end up at the non-Pareto optimal outcome of $(1, 0)$ at AC.

2. The following $2 \times 2$ nonzero-sum games each have at least one pure-strategy Nash equilibria in which Rose does not receive her maximum payoff. Decide if any of the following strategic moves will get Rose her best achievable payoff: seizing the first move, forcing Colin to make the first move, making a threat to Colin and forcing Colin to make the first move, or making a promise to Colin and forcing Colin to make the first move. (Remember that, as we discussed in class, Rose won't be able to go for her BEST possible payoff if it coincides with Colin's WORST possible payoff.) If your answer is a threat or promise, also describe how Rose could implement it by lowering one of her own payoffs before the game.

(a)

| | A | B |
|---|---|---|
| A | (-2,3) | (1,2) |
| B | (0,1) | (4,0) |

**Solution:** Rose would like to achieve her best payoff of 4 at BB, but she cannot do this via a strategic move since it would force Colin to accept his WORST possible payoff (and as we discussed in class, this is impossible.) So instead she will shoot for her second best payoff of 1 at AB.

She can get this by making the following promise: "Colin, if you play B, I will play A (rather than B, which would be my natural response!)."

This puts Colin in the following position: if he plays A, then Rose responds with her best option B (she made no promise in this case, after all), giving them the payoffs $(0, 1)$. If he plays B, then Rose responds with A (as per her promise), giving them the payoffs $(1, 2)$. Of these two, Colin prefers his payoff of 2 at $(1, 2)$. So, he will play B (as Rose wants), and Rose gets the outcome of $(1, 2)$ that she wanted.

Rose could make this promise completely credible by lowering her payoff at BB to any number lower than 1, for instance $-1$. Then, A would actually be her best response to Colin B.

(b)

| | A | B |
|---|---|---|
| A | (2,3) | (4,2) |
| B | (1,1) | (0,5) |

**Solution:** Rose would like to achieve her best payoff of 4 at AB, and she should be able to do this since 2 is not Colin's worst payoff. (His worst payoff is 1, at BA.)

She can get this by making the following threat: "Colin, if you play A, I will play B (rather than A, which would be my natural response!)."

This puts Colin in the following position: if he plays A, then Rose responds with B (as per her threat), giving them the payoffs $(1, 1)$. If he plays B, then

Rose responds with her best response A (she made no promise in this case, after all), giving them the payoffs $(4, 2)$. Of these two, Colin prefers his payoff of 2 at $(4, 2)$. So, he will play B (as Rose wants), and Rose gets the outcome of $(4, 2)$ that she wanted.

Rose could make this threat completely credible by lowering her payoff at AA to any number lower than 1, for instance $-1$. Then, B would actually be her best response to Colin A.