FSEM Homework Assignment 8

Show your work!

1. For each of the following positions in a game of STANDARD Nim (i.e. not limited; you can take as many chips as you want from any pile, person to take the last chip wins), use binary expansions of the pile numbers to decide if the position is a W(in) or L(os) for the player about to move. For the positions that are W(in), list ALL winning moves, i.e. moves which present the next player with a L(osing) position.

   (a) Three piles, with 4, 7, and 8 objects.

   (b) Three piles, with 21, 42, and 63 objects.

   (c) Four piles, with 20, 25, 26, and 28 objects.

   (HINT: in case it's helpful, here is a list of the powers of 2 that you'll need: \(2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32\).)

2. We discussed in class how certain “holes” in the game of Cram are equivalent to positions in Nim. For example, \(\begin{array}{|c|c|c|c|c|c|} \hline \times & \times & \times & \times & \times & \times \end{array}\) is obviously equivalent to a Nim pile with 1 chip in it: no matter what move is made, there is no room for another domino afterwards, so the position is like a single “chip” which can be taken.

   Slightly more tricky is \(\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline \times & \times & \times & \times & \times & \times & \times & \times & \times \end{array}\). Here, there are two possible types of moves: you could fill the leftmost two squares, leaving space for one domino (which is like a pile with 1 chip), or fill the middle two squares, leaving no space for any dominoes (which is like a pile with 0 chips.) Since the legal moves leave either 1 chip or 0 chips, \(\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline \times & \times & \times & \times & \times & \times & \times & \times & \times \end{array}\) is like a pile with 2 chips (just think of the legal moves in Nim from a pile with 2 chips...)

   Continuing with this idea of working backwards to equate holes in Cram to Nim positions, explain what Nim positions the following are equivalent to:

   (a) \(\begin{array}{|c|c|} \hline \times & \times \end{array}\)

   (b) \(\begin{array}{|c|c|c|c|c|c|} \hline \times & \times & \times & \times & \times & \times \end{array}\)

3. You’re playing a game of Cram, and after your opponent’s move, are presented with the following position. Use reasoning as in problem 2 to describe an equivalent Nim position (it will have lots of piles!) and determine a winning move.

\(\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \end{array}\)