

FSEM Midterm Exam

Instructions: Answer every question as completely as possible and **SHOW ALL WORK**. Make sure that your final answers are clearly marked where applicable. You may use the backs of the test sheets, and if you need additional scratch paper, just ask. Calculators should not be necessary for any exam question, but a basic scientific calculator (no graphing or programmable capabilities) may be used. Cell phones **CANNOT BE CONSULTED** during the exam for any reason. Your exam should consist of four problems, worth a total of 50 points.

1. Solve the following zero-sum matrix games. To be clear, this means that you should present (possibly mixed) strategies for **BOTH** players and a value v so that Rose's strategy guarantees her an expected payoff of at least v , and Colin's strategy guarantees that her expected payoff is at most v . To save time, **MAKE SURE** that you begin by looking for saddle points and/or dominated strategies!!!

(a)

	A	B	C
A	1	-2	-5
B	0	-1	3
C	2	-4	-1
D	-4	-3	4

(b)

	A	B	C	D
A	-4	0	-3	3
B	0	-2	1	-1
C	5	-1	1	4

2. For the game from part (a) of problem 1, figure out the optimal strategies in the following cases. (Your answers do not need to me mixed strategies here!!!)

(a) Rose's best counterstrategy if Colin is playing B every time

(b) Rose's best counterstrategy if Colin is playing all three of his options with equal probability $\frac{1}{3}$

(c) Colin's best counterstrategy if Rose is playing A $\frac{2}{3}$ of the time and B $\frac{1}{3}$ of the time

3. For the following nonzero-sum game,

- (i) Draw the movement diagram to find all pure-strategy Nash equilibria
- (ii) Draw the payoff polygon and highlight all Pareto optimal outcomes
- (iii) State whether the game is solvable in the strict sense (SSS)
- (iv) Find a strategic move for Rose, such as seizing first move, making Colin move first and threatening, or making Colin move first and promising, that guarantees her the best achievable payoff

	A	B
A	(4,4)	(2,5)
B	(5,2)	(0,0)

4. Write the payoff matrix (using expected payoffs) for the game of Final Jeopardy!, as discussed in class, in the case where Rose has \$2 and Colin has \$1. DO NOT ATTEMPT TO SOLVE THE RESULTING GAME; JUST WRITE THE MATRIX. Just to make sure the question is clear, I will summarize how this game works: (however, if you were in class, you shouldn't need to read this) each player makes a wager anywhere from \$0 to their total, then the players are asked a Final Jeopardy! question, which each player has a 50% probability of answering right and a 50% probability of answering wrong. Each player has their wager added or deducted from their score based on whether they got the question right or wrong, and the final totals are compared. If Rose has more, she wins and gets a payoff of 1, if Colin has more, he wins and Rose gets a payoff of -1 , and if they tie, Rose gets a payoff of 0.

5. Short answer: answer each of the following questions and briefly explain your answer in your own words. Your answers should not need to be any longer than a few sentences. You are not necessarily supposed to know the answers to these questions offhand; they can require thinking creatively about topics from the course.

(a) Explain why the removal of dominated strategies can be dangerous in a non-zero sum game.

(b) In your own words, explain what a Nash equilibrium is.

(c) Suppose you analyze a zero-sum game, and find out that the value is -2 . Who has the advantage in this game? Is there a way to make a game where the strategies for the players should be exactly the same, but the game is “fair” (i.e. has value 0)?

(d) If Rose and Colin are playing Final Jeopardy! with the assumptions from our class, and Rose has \$10000 and Colin has \$6000, what are optimal wagers for both players? (Hint: we’re in the case where Rose has between 1.5 times Colin’s starting amount and 2 times Colin’s starting amount.) NOTE: you do not have to completely explain your answer, though I will give extra credit for good explanations.