Solutions

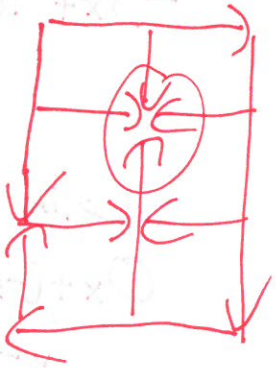
FSEM Midterm Exam

Instructions: Answer every question as completely as possible and SHOW ALL WORK. Make sure that your final answers are clearly marked where applicable. You may use the backs of the test sheets, and if you need additional scratch paper, just ask. Calculators should not be necessary for any exam question, but a basic scientific calculator (no graphing or programmable capabilities) may be used. Cell phones CANNOT BE CONSULTED during the exam for any reason. Your exam should consist of four problems, worth a total of 50 points.

1. Solve the following zero-sum matrix games. To be clear, this means that you should present (possibly mixed) strategies for BOTH players and a value v so that Rose's strategy guarantees her an expected payoff of at least v, and Colin's strategy guarantees that her expected payoff is at most v. To save time, MAKE SURE that you begin by looking for saddle points and/or dominated strategies!!!

(a)

	A	B	C
A	1	-2	-5
B	0	-1	3
C	2	-4	-1
D	-4	-3	4



saddle pt/ Nash equilibrium at BB

Rose, Colin should each play B; value of game is (-1)

(b)

	A	B	C	D
A	-4	0	-3	3
~~B~~	~~0~~	~~-2~~	~~1~~	~~-1~~
C	5	-1	1	4

Rose B domin. by C

Then,

Colin D dom. by B

	A	B	C
A	-4	0	-3
C	5	-1	1

Set up Rose's mixed strat.

x = prop. of time Rose plays A

$1-x$ = prop. of time Rose plays C

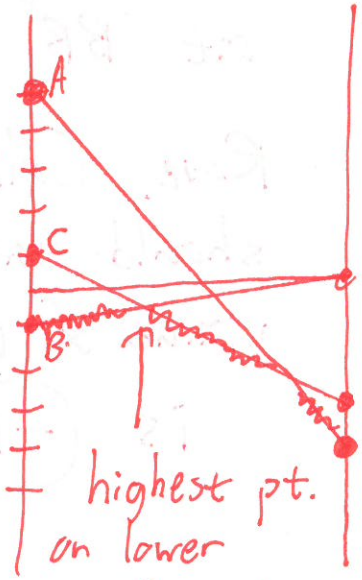
Colin counterstrats:

Colin A: exp. payoff $-4x+5(1-x)$

B: " $0x+(-1)(1-x)$

C: " $-3x+1(1-x)$

Graph:



highest pt. on lower envelope is crossing of B, C

Colin B = Colin C

$0x+(-1)(1-x) = -3x+1(1-x)$

$x-1 = -3x+1-x$

$5x = 2$

$x = \frac{2}{5}$

exp. payoff: plug into Colin B or C

$x-1 = \frac{2}{5}-1 = -\frac{3}{5}$

Rose plays A $\frac{2}{5}$ of time,
C $\frac{3}{5}$ of time

On average, she loses $\frac{3}{5}$ per game

Now, we know Colin will play only B, C

	B	C
A	0	-3
C	-1	1

x = prop. of time Colin plays B

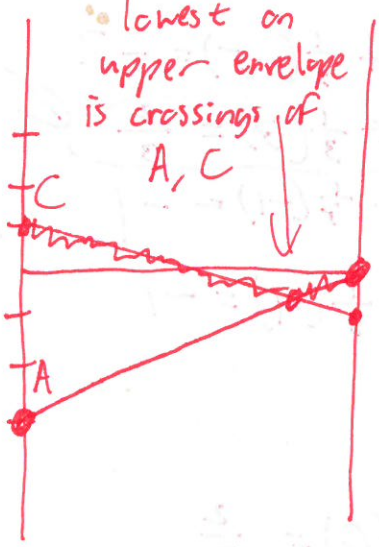
$1-x$ = " " " " " C

Rose's counterstrats:

Rose A: exp payoff $0x + (-3)(1-x)$

C: " " $-1x + 1(1-x)$

Graph:



Rose A = Rose C

$$0x + (-3)(1-x) = -1x + 1(1-x)$$

$$-3 + 3x = -x + 1 - x$$

$$5x = 4$$

$$x = \frac{4}{5}$$

exp. payoff: plug in

$$-3 + 3\left(\frac{4}{5}\right) = \frac{-3}{5}$$

Colin should play B $\frac{4}{5}$ of time, C $\frac{1}{5}$ of time

Value of game = $\boxed{\frac{-3}{5}}$

Use expected payoffs

2. For the game from part (a) of problem 1, figure out the optimal strategies in the following cases. (Your answers do not need to me mixed strategies here!!!)

(a) Rose's best counterstrategy if Colin is playing B every time

versus Colin B:

Rose A: -2

B: -1 ← Rose's best option

C: -4

D: -3

(b) Rose's best counterstrategy if Colin is playing all three of his options with equal probability $\frac{1}{3}$

versus Colin $\frac{1}{3}$ A/$\frac{1}{3}$ B/$\frac{1}{3}$ C:

Rose A: $\frac{1}{3}(1)+\frac{1}{3}(-2)+\frac{1}{3}(-5)=-2$

B: $\frac{1}{3}(0)+\frac{1}{3}(-1)+\frac{1}{3}(3)=\frac{2}{3}$ ← Rose's best option

C: $\frac{1}{3}(2)+\frac{1}{3}(-4)+\frac{1}{3}(-1)=-1$

D: $\frac{1}{3}(-4)+\frac{1}{3}(-3)+\frac{1}{3}(4)=-1$

(c) Colin's best counterstrategy if Rose is playing A $\frac{2}{3}$ of the time and B $\frac{1}{3}$ of the time

versus Rose $\frac{2}{3}$ A/$\frac{1}{3}$ B:

Colin A: $\frac{2}{3}(1)+\frac{1}{3}(0)=\frac{2}{3}$

B: $\frac{2}{3}(-2)+\frac{1}{3}(-1)=\frac{-5}{3}$

C: $\frac{2}{3}(-5)+\frac{1}{3}(3)=\frac{-7}{3}$ ← Colin's best option

4. Write the payoff matrix (using expected payoffs) for the game of Final Jeopardy!, as discussed in class, in the case where Rose has \$2 and Colin has \$1. DO NOT ATTEMPT TO SOLVE THE RESULTING GAME; JUST WRITE THE MATRIX. Just to make sure the question is clear, I will summarize how this game works: (however, if you were in class, you shouldn't need to read this) each player makes a wager anywhere from \$0 to their total, then the players are asked a Final Jeopardy! question, which each player has a 50% probability of answering right and a 50% probability of answering wrong. Each player has their wager added or deducted from their score based on whether they got the question right or wrong, and the final totals are compared. If Rose has more, she wins and gets a payoff of 1, if Colin has more, he wins and Rose gets a payoff of -1, and if they tie, Rose gets a payoff of 0.

	0	1
0	1	$\frac{1}{2}$
1	$\frac{1}{2}$	$\frac{1}{2}$
2	0	$\frac{1}{4}$

Rose bets 1
Colin bets 1

Rose correct
$2+1=3$
Rose wins: +1
$1+1=2$
Colin correct

Rose incorrect
$2-1=1$
Rose wins: +1
$1-1=0$
Colin incorrect

Rose wins: +1

Rose loses: −1

Exp. payoff: $\frac{1}{4}(1)+\frac{1}{4}(1)+\frac{1}{4}(-1)+\frac{1}{4}(1)=\frac{1}{2}$

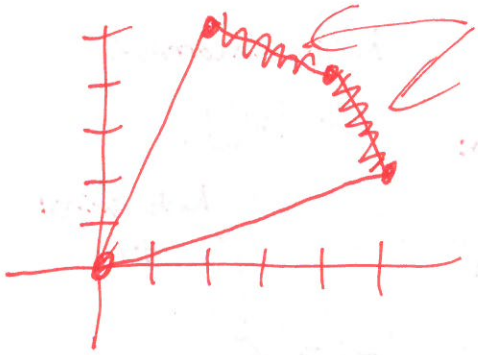
Other five entries of matrix are similar

3. For the following nonzero-sum game,

(i) Draw the movement diagram to find all pure-strategy Nash equilibria
(ii) Draw the payoff polygon and highlight all Pareto optimal outcomes
(iii) State whether the game is solvable in the strict sense (SSS)
(iv) Find a strategic move for Rose, such as seizing first move, making Colin move first and threatening, or making Colin move first and promising, that guarantees her the best achievable payoff

	A	B
A	(4,4)	(2,5)
B	(5,2)	(0,0)

(i) NE at AB, BA

(ii) 

(iii): NOT SSS: the NE are both PO, and do not have same payoffs (also don't form a grid)

(iv): Rose wants her 5 payoff at BA. Gets it by forcing Colin to choose btwn BA, BB, for instance by taking first move and playing B

5. Short answer: answer each of the following questions and briefly explain your answer in your own words. Your answers should not need to be any longer than a few sentences. You are not necessarily supposed to know the answers to these questions offhand; they can require thinking creatively about topics from the course.

(a) Explain why the removal of dominated strategies can be dangerous in a non-zero sum game.

For instance, in Prisoner's Dilemma,

~~(-1, -1)~~	~~(-10, 0)~~
(0, -10)	(-5, -5)

Removing dom. strats leads to (-5, -5) which is a bad outcome for the group, for instance it's not Pareto optimal ((-1, -1) is better for both!)

(b) In your own words, explain what a Nash equilibrium is.

It's a location all arrows point into on a movement diagram

OR

It's a place in payoff matrix where neither player has any incentive to move given other player's strategy

(there are other reasonable answers as well)

(c) Suppose you analyze a zero-sum game, and find out that the value is -2. Who has the advantage in this game? Is there a way to make a game where the strategies for the players should be exactly the same, but the game is "fair" (i.e. has value 0)?

If both players play optimally, on average Colin will win 2 every game, and he has the advantage.

To make a game w/same strategies which is fair, add 2 to each payoff.

(d) If Rose and Colin are playing Final Jeopardy! with the assumptions from our class, and Rose has \$10000 and Colin has \$7000, what are optimal wagers for both players? (Hint: we're in the case where Rose has between 1.5 times Colin's starting amount and 2 times Colin's starting amount.) NOTE: you do not have to completely explain your answer, though I will give extra credit for good explanations.

Rose should bet $2C - R + 1 =$ 2,001
Colin should bet it all: 6,000

If Rose gets it right, she has \$12,001 and guarantees victory, Even if she is wrong she has \$7,999 and will win if Colin gets question wrong. Rose therefore wins in 3 of 4 cases, which is the best she can do (Colin can always win at least if he doubles his money & Rose is wrong). So, this is a saddle pt/Nash equilibrium.