## FSEM Practice Midterm

**Instructions:** Answer every question as completely as possible and SHOW ALL WORK. Make sure that your final answers are clearly marked where applicable. You may use the backs of the test sheets, and if you need additional scratch paper, just ask. Calculators should not be necessary for any exam question, but a basic scientific calculator (no graphing or programmable capabilities) may be used. Cell phones CANNOT BE CONSULTED during the exam for any reason. Your exam should consist of four problems, worth a total of 50 points.

1. Solve the following zero-sum matrix games. To be clear, this means that you should present (possibly mixed) strategies for BOTH players and a value v so that Rose's strategy guarantees her an expected payoff of at least v, and Colin's strategy guarantees that her expected payoff is at most v. To save time, MAKE SURE that you begin by looking for saddle points and/or dominated strategies!!!

(a)

	A	В	$\mathbf{C}$
Α	2	1	3
В	4	-1	-2
$\mathbf{C}$	-3	0	4

	A	В	$\mathbf{C}$
Α	3	-3	1
В	-1	3	-2
$\mathbf{C}$	2	0	1

(b)

2. For each of the following nonzero-sum games,

(i) Draw the movement diagram to find all pure-strategy Nash equilibria

(ii) Draw the payoff polygon and highlight all Pareto optimal outcomes

(iii) State whether the game is solvable in the strict sense (SSS)

(iv) Find a strategic move for Rose, such as seizing first move, making Colin move first and threatening, or making Colin move first and promising, that guarantees her the best achievable payoff (note: some of her payoff(s) might not be achievable because Colin won't ever make a choice which lets her have them!)



	A	В
Α	(-2,2)	(-1,-1)
В	(-3,-3)	(2,-2)

	A	В
А	(3,2)	(-4,5)
В	(5,-4)	(-2,-3)

(b)

**3.** Consider the following game: Rose chooses either the number 1 or the number 5, and Colin chooses either the number 2 or the number 4. Then, a fair die is rolled, which shows a random number from 1 to 6. The person whose number is closest to the roll of the die wins \$1 from the other player. (So this is a zero-sum game!) As an example, say Rose chooses 5 and Colin chooses 2. The die is rolled, and comes up a 6. In this case, Rose would win, since 6 is closer to 5 than it is to 2.

Write down the payoff matrix for this game, using EXPECTED payoffs in each case. Do not try to solve the resulting game, just write the matrix.

4. Short answer: answer each of the following questions and briefly explain your answer in your own words. Your answers should not need to be any longer than a few sentences. You are not necessarily supposed to know the answers to these questions offhand; they can require thinking creatively about topics from the course.

(a) We never discussed solutions of  $1 \times 2$ ,  $1 \times 3$ ,  $1 \times 4$ ,  $2 \times 1$ ,  $3 \times 1$ ,  $4 \times 1$ , etc. zero-sum games in this class. Can you explain why the study of such games would be quite easy?

(b) Suppose that in some zero-sum game where you have two strategies, A and B, you determine your optimal mixed strategy is to play A  $\frac{1}{3}$  of the time and B  $\frac{2}{3}$  of the time. Why is it not a good idea to implement this by just playing A, B, B, A, B, B, A, B, B, ...? Give a specific example of a better way to implement this mixed strategy.

(c) We never discussed the idea of Pareto optimal outcomes in zero-sum games; this is because the idea is very uninteresting for zero-sum games. Why is this?

(d) In your own words, explain why the following is true: if a zero-sum game has a saddle/equilibrium point, it is a natural and logical strategy for both players. (There are several "right" answers to this question.)